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THE UNIVERSITY OF ALBERTA

INTERPRETATION OF LONG AEROMAGNETIC PROFILES

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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ABSTRACT

In this thesis an investigation is made of the properties of long aeromagnetic profiles in order to determine some aspects of the magnetic character of the crust of the Earth. In particular, an interpretation is made of those broad anomalies whose widths lie between 100 and 150 miles. These broad anomalies are extracted from the profile data using numerical filtering. The frequency characteristics of the filters to be applied are determined by performing a power spectral analysis on the data. The spectral analyses also provide some insight as to the magnetic character of the crust.

The many laborious calculations involved in such an investigation have been evaluated with the aid of the IBM 1620 digital computer in the Computing Centre, University of Alberta.

The broad anomalies revealed by numerical filtering are interpreted as being due to anomalous Curie point geotherm structure and estimates of the depth to the geotherm and of susceptibility contrast at these depths are made for both the continental and oceanic crusts.

The results suggest that:

- (i) the Curie point depth for the continental crust is slightly greater than that for the oceanic crust,
- (ii) the geotherm depth is very close to that of the Mohorovicic discontinuity for the continental crust and much greater than that of the discontinuity for the oceanic crust,
- (iii) the susceptibilities of rocks at the geotherm depth are greater for the continental crust.

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I. INTRODUCTION

1.1 The Problem

In many recent geophysical publications (Dobrin and Ward, 1962; Goldstein, 1962; Dean, 1958) it has been suggested that statistical analysis and the application of filtering to geophysical data might reveal a new technique in the interpretation of many problems, especially in problems of potential field signals. Although the mathematics for such operations have been known for two or three decades, the laborious calculations required for such interpretation were never thought to be practical and too, there were those, as there still are today, who felt that potential field problems do not lend themselves to such an analysis. With the advent in recent years of high-speed analog and digital computers much interest and research has been initiated along these lines in hope that some light might be shed upon problems whose solutions have escaped other interpretational procedure or have lacked corroboration by a more detailed type of analysis.

One such problem has been that of the Curie point geotherm, the depth to which is very important in the interpretation of deep magnetic sources. Such information could also give the interpreter an insight as to the rock type which occurs at the geotherm depth.

In the past, the Curie point geotherm has always been interpreted as the lower boundary of magnetic sources which give rise to the broader features of magnetic maps or profiles. In this thesis it is shown that certain broad anomalies of an aeromagnetic profile (with dipole field removed) as revealed by spectral analysis and numerical filtering could be due to upwarping or downwarping of the geotherm. The analysis is performed on data from both continental and oceanic aeromagnetic profiles and it is hoped that some insight will be gained into the difference between continental and oceanic magnetic crustal character as well as the nature of the Curie point geotherm.

1.2 Terrestrial Magnetism

The existence of a magnetic field associated with the earth was discovered as early as the eleventh century when mariners began using the magnetic compass in navigation. However, it was not until 1600, that the true nature of the magnetic field, as observed at the earth's surface, was revealed by the experimental work of William Gilbert. His publication "De Magnete" pointed out that the earth itself acts to a certain extent as though it were a great spherical magnet. Measurements of the magnetic field and its variations with time have been made since 1600 and "Terrestrial Magnetism" may therefore be described as one of the oldest fields in geophysics.

Investigation of the earth's magnetic field utilizing potential theory has led certain observers to believe that it originates from sources inside the earth, outside the earth, and on the surface of the earth itself.

By applying spherical harmonics to observations of the field made at the earth's surface it has been found that 94 per cent of the field originates from sources inside the earth. Furthermore the observed internal field can be described as the effect of a number of fictitious magnetic dipoles, each with a different orientation, located at the centre of the earth. The first of these imaginary dipoles is located with its axis along the magnetic axis of the earth whose poles are displaced by approximately 18° of latitude. Because they are not diametrically opposite one another the poles are joined by a line which passes some 750 miles from the earth's centre. Because of the irregularities in the earth's magnetic field, as observed at the surface, an indefinite number of weaker dipoles with different orientations are added; their various strengths and orientations are determined from global measurements of the field such that the best possible fit to all the data is obtained. However, if one assumes the earth to be uniformly magnetized and that only the axially oriented dipole is present then the value of polarization required to account for the earth's field is many orders of magnitude higher than that of igneous rocks associated with the earth's crust. This,

coupled with the fact that as the temperature of a permanently magnetized material is increased it loses its ability to become permanently magnetized, make this theory of imaginary dipoles largely untenable since the Curie points¹ of all magnetic materials are reached at very shallow depths below the earth's surface.

Experimentation on the effect of increased pressure on the Curie point of some materials shows that the Curie temperature can be increased but in general it is believed that the change would not be sufficient to make permanent magnetization possible within the depths of the earth.

The most widely held theory as to the origin of the internal field is that it is due to motions within the liquid core. Two possible causes of motion within the core have been proposed. The first ascribes it to the variation in the angular velocity of the earth due to tidal friction and the second is derived from a temperature gradient within the core sufficiently large enough to produce thermal convection. Because the first effect is extremely small it has been ruled out as a possible cause of the required motion. Upon acceptance of the second cause then speculation was made as to the mechanism under which motions in the core might set up electromotive forces.

¹ Curie point - the temperature at which a substance upon being heated loses its ferromagnetic properties and begins to behave as a paramagnetic substance if placed in a magnetic field.

Again two ways have been put forward. The first is that the motions set up temperature distributions within the core giving rise to thermoelectric currents. According to Runcorn (1956) potential differences across a contact between two spherical surfaces of dissimilar materials or similar materials under different physical conditions will excite the electric mode. Runcorn further states that a potential difference of 1 volt is sufficient to produce sufficient electric mode magnetic field. Quantitative results are not available, however, for this theory at present.

The second means by which motion might set up electromotive forces in the core is known as the dynamo theory. This theory suggests that the field is produced and maintained by an induction mechanism with the kinetic energy of the fluid motions in the core being converted to magnetic energy. The crux of the objection to this hypothesis is whether a sphere, possessing asymmetric fluid motion, but unlike a real dynamo not possessing asymmetry of structure can in fact be a self-exciting dynamo independent of a sustaining field from an external source. Bullard and Gellman (1954), using one particular set of motions, have devised a solution; the convergence of this solution however is debatable. Nevertheless, there seems to be a general consensus of opinion that motions in the fluid core are primarily responsible for the internal magnetic field of the earth.

Only a small percentage of the earth's field need be accounted for by an external field. Most of the theories proposed to explain the source of the external field involve the inductive effect of circulating electric currents in the ionosphere.

The component of the earth's field purported to lie on the surface of the earth and sometimes called the non-potential field is supposedly due to electric currents flowing from the earth to the atmosphere and back again across the earth's surface. However, many feel that the residue of field for which this theory attempts to account is actually due to errors in observation and not to any source at all.

The earth's magnetic field has associated with it variations with time in both strength and direction. These variations can be categorized under two main types: secular variations or changes in the earth's field progressing over decades or centuries and diurnal variations or daily changes which are smaller but more rapid in oscillation. A very comprehensive treatment of these variations can be found in Chapman and Bartels treatise on geomagnetism.

1.3 Magnetization in the Crust

Having described the terrestrial magnetic field and its possible origin an examination will now be made of the effect of this field on the magnetically susceptible materials

in the earth's crust. With respect to local areas, the earth's magnetic field can be said to be uniform over areas of magnetically homogeneous composition in the crust, but is measurably distorted in regions of inhomogeneous composition. Most of these magnetic anomalies are due to igneous rocks, sedimentary rocks which contain iron oxides and to some extent concentrations of magnetic iron ores. The strength of an anomaly generally depends on the susceptibilities¹ of the materials in the subsurface structure and also on the extent of the structure.

Generally, the magnetic character of a rock can be considered to be made up of residual and induced magnetization. Induced magnetization is that which is acquired by a rock due to induction by the earth's present magnetic field. Residual or permanent magnetization is that acquired by the rock as it cooled, during its molten state, through its Curie temperature. Such magnetization is termed permanent because it is highly resistive to removal or reversal. It is frequently appreciable in magnitude and may not be in the same direction as the induced magnetization associated with the rock.

Thus the magnetic character of the crust is essentially superimposed on that of the geomagnetic field

¹ susceptibility - a measurement of the ability of a substance to be magnetized under the influence of a magnetic field.

and any attempt to analyze the field associated with a portion of the crust must involve the removal of the regional gradient due to the terrestrial field.

The magnetic character of the crust varies laterally in response to changes in content of ferromagnetic materials. However, in relation to depth below the surface two other factors, temperature and to a certain extent pressure, govern the amount of magnetization. The temperature gradient in the crust is approximately 30° C/km, so that at a depth of about 25 km., the Curie point for iron, namely 750° C., is reached. At the Curie point ferromagnetic materials become paramagnetic and essentially lose most of their detectable magnetic character. The depth extent of magnetization within the crust is thus temperature controlled; the influence of pressure in raising the Curie point of a material has not yet been fully determined but more will be said about this in the next section.

1.4 Review of Publications on Related Topics

In 1941, Vacquier and Affleck made a statistical study of some 126 local magnetic anomalies in an effort to estimate the average depth to the Curie point geotherm. They assumed the crust to be divided into three distinct layers: the first being a sedimentary layer of from zero to ten miles in thickness, a magnetic granitic layer and

an underlying layer of non-magnetic or homogeneously magnetic material. Using the narrow anomalies to calculate the depth to the top of the granitic layer, curves, for intensity of magnetization versus this depth with depth to the bottom of granitic layer or to the bottom of the layer of inhomogeneous magnetic material as a parameter, were constructed. The data were then fitted to these curves with the best fit result giving an average depth to the base of the inhomogeneities of 12.5 miles. Now density contrasts at much greater depths are indicated by gravity data and therefore inhomogeneity in magnetic content should occur if the material at these depths can be magnetized. However, Vacquier and Affleck found no anomalies which indicated a source at a greater depth and therefore they concluded that the base of the inhomogeneous layer coincided with the Curie point geotherm.

The above estimate of the depth falls within the range of those values predicted for the depth to the bottom of the granitic layer by Benfield (1940). His work was based on a mathematical treatment of heat flow measurements. However, the base of the inhomogeneous magnetic layer which Vacquier and Affleck consider to be the Curie point geotherm could be the Conrad discontinuity. The depth estimate of 12.5 miles is within the range of the depth to the Conrad of 5 - 20 miles (Gutenberg, 1955).

In 1955, Birch published a paper in which he

commented on the work of Vacquier and Affleck mentioned above. He pointed out that they may have excluded broader anomalies resulting from sources at greater depths. His scepticism with respect to their results seems to result from his estimates of temperature versus depth within an average crust. For various models of crustal structure embodying varied radioactive content and heat flow contribution from below the crust and also due to radioactive decay Birch constructed temperature - depth curves which approach the Curie point for magnetite (578°C) in the 30 to 40 km. range, which is from $1\frac{1}{2}$ to 2 times greater than the estimate of Vacquier and Affleck. Birch also states that ignorance of the magnetic character of the composition of the rocks at these depths raises the question as to whether further analysis of regional magnetic anomalies might not shed some light on this subject.

Serson and Hannaford (1957) published a paper in which they gave their results from a statistical analysis of aeromagnetic profiles flown over continental and oceanic regions in 1955. They calculated autocovariance functions for profiles flown over Western Canada and the Atlantic Ocean and then fitted models to these curves. They concluded that oceanic magnetic character was made up of poles at a depth of one kilometre below the ocean floor with an intensity (susceptibility) of at least 0.005 c.g.s. units. Furthermore, if these poles had corresponding poles their depth has to be greater than 30 km. (that is a layer of dipoles $\geq$ 30 km.

in thickness). Under the continents, they said the rocks must have a susceptibility of 0.05 c.g.s. or greater (for Western Canada) and that the magnetic structure consisted of a thin layer at a depth of 10 to 12 km. or a layer from 4 km. downwards.

Perhaps the most enlightening contribution on the use of aeromagnetic profiles to probe into the earth's magnetic character was published by Alldredge and Van Voorhis in 1961. Using profiles with a length of at least 2000 miles and no harmful heading changes they removed that part of the field due to an axially centred dipole and then plotted a histogram of crossover distances. This, in effect, was a very simple type of harmonic analysis. The histogram showed two definite ranges for crossover distances, those with a range of 0 to 260 nautical miles and those in the range 2100 to 5200 nautical miles. They attributed the former to crustal magnetic sources and the latter to sources near the core-mantle boundary. The large gap between the crossover distance ranges seems to indicate that the mantle is non-magnetic in character. As pointed out by Serson and Hannaford also, Alldredge and Van Voorhis found that the majority of anomalies occurred in the short-wavelength category. In their case they found that 93 per cent had a crossover distance of 60 nautical miles or less.

1.5 Object of the Thesis

The purpose of this thesis, in relation to the statement of the problem as outlined in section 1.1, is three-fold:

- (1) To gain an insight into contrasting or similar features of the magnetic character of continental and oceanic crusts by means of power spectral analysis on aeromagnetic data.
- (2) To show that numerical filtering can be effectively applied to geophysical data to reveal information which was hitherto inaccessible through ordinary interpretational procedures.
- (3) To show that certain anomalies revealed by numerical filtering can be interpreted as anomalous Curie point geotherm structure and thus perhaps outline a practical method for detailed analysis of magnetic features due to deep sources.

II. POWER SPECTRA

2.1 Introduction

Allredge and Van Voorhis (1961) found that when they performed a harmonic analysis on global magnetic data the resulting spectrum revealed two possible sources of terrestrial magnetic anomalies. One was the core-mantle boundary with which the dipole field was associated and the other was the magnetic material in the crust.

If the dipole field, which is composed of long period signals, were removed, then the shorter period crustal anomalies could be studied more reliably with respect to a base line from which the long period trend had been removed. It has been speculated that the anomalies associated with the Curie point geotherm, if they exist, would be broad and comparatively small in amplitude due to the depth of the geotherm (approximately 10-60 miles below the surface; Serson and Hannaford, 1957). Because of these two characteristics, the anomalies are likely to be slightly masked by those of near-surface magnetic materials. Nevertheless, the energy associated with these broad anomalies ought to be comparatively greater than that of the narrower near - surface structures. For this reason and because of the fact that a power spectrum is also a function of the frequency of occurrence of a certain period, a power spectrum analysis was performed on the magnetic

data available. It was hoped that this analysis might provide a clearer picture of the type of anomalies present in a typical magnetic profile, as well as perhaps, to define more clearly some long period anomalies whose origins might be attributed to the Curie point geotherm.

Because the mathematical approach to spectrum analysis is generally based on communications engineering, one seems to associate such an analysis with a stationary time series, that is one in which frequencies are expressed in cycles/unit time. However, the spectrum analysis is equally applicable to a stationary space series and one need only substitute the space variable for the time variable, thereby, for example, obtaining frequencies in cycles/unit length.

If a measured signal¹ containing noise can be described as an approximately stationary random process then the associated autocovariance function or power spectrum will generally be the most useful display for determining the presence of periodicities, according to Blackman and Tukey (1958).

The power spectrum is nothing more than the Fourier transform of the autocovariance function; however,

signal¹ - The signal, as referred to in this thesis, shall be defined as a message which is recorded on a strip chart and upon which is superimposed noise, both instrumental and near-surface geological noise.

when these functions must be determined by measurement and computation and then interpreted, the power spectrum lends itself to application more readily than the autocovariance function. The reason is that the criteria developed so far for the application of autocovariance analysis are so complicated that it is very difficult to apply them in practice.

2.2 Definitions

For an ideal stationary Gaussian process the autocovariance function is defined by

$$C(\lambda) = \lim_{X \rightarrow \infty} \frac{1}{X} \int_{-X/2}^{X/2} H(x) \cdot H(x+\lambda) dx$$

where x is the space variable

and λ is the lag in units of length.

$C(\lambda)$ may be rewritten as

$$C(\lambda) = \int_{-\infty}^{\infty} P(f) \cdot e^{i2\pi f\lambda} df$$

$$\text{where } P(f) = \lim_{X \rightarrow \infty} \frac{1}{X} \left| \int_{-X/2}^{X/2} H(x) \cdot e^{-i2\pi fx} dx \right|^2$$

and $P(f)$ is the power spectrum of the process considered.

The power spectrum can also be expressed as the Fourier transform of the autocovariance function, that is

$$P(f) = \int_{-\infty}^{\infty} C(\lambda) e^{-i2\pi f\lambda} d\lambda$$

Since both $P(f)$ and $C(\lambda)$ are even functions and because only positive frequencies are of interest we have that

$$P(f) = 2 \int_0^{\infty} C(\lambda) \cos(2\pi f \lambda) d\lambda$$

and
$$C(\lambda) = 2 \int_0^{\infty} P(f) \cos(2\pi f \lambda) df$$

2.3 Derivation of Power Spectra for Equally Spaced Records

In practice we can obtain only a limited number of pieces of $H(x)$ of finite length; each piece can be regarded as a sample of the population of pieces $H(x)$ of the same length. The reduction of such data will therefore only produce estimates of the power density spectrum and these will be subject to sampling variations and to biases in the usual statistical sense. The latter statement is especially true of records for which $H(x)$ is obtainable only at uniformly spaced values of x within each piece $H(x)$. Theoretical study of sampling variability and bias shows, however, that estimates of power within two narrow frequency bands separated by a small, fixed separation will become more precise and will approach statistical independence with respect to each other as we consider longer and longer records.

One of the inherent difficulties in using equally spaced data is that of aliasing of frequencies. Suppose $H(x)$ is available only for equally spaced values of x , say

$$x = 0, \Delta x, 2\Delta x, \dots, n\Delta x.$$

then it is clear that the highest frequency which can be defined by such a sampling interval is

$$f_N = \frac{1}{2\Delta x}$$

where f_N is the folding (or Nyquist) frequency. Then the frequencies f , $2f_N - f$, $2f_N + f$, $4f_N - f$, $4f_N + f$, and so on, are aliases of one another where f is the principal alias. However, with the data used for this thesis the problem of aliasing can be neglected because of the altitude at which measurements were made. The sampling interval employed was three miles and the folding frequency is therefore $1/6$ cycles/mile which is undoubtedly larger than the highest detectable frequency at the altitudes at which the records were made.¹ The fortunate exclusion of the problem of aliasing makes the spectral estimates more reliable than they would be ordinarily.

We proceed now to outline a method for power spectrum estimation from a uniformly spaced discrete space series of finite length; this outline is derived from the more complete discussion given by Blackman and Tukey.

First, let $H_0, H_1, \dots, H_n$ be the space series with a sampling interval Δx .

¹To see this, the reader is referred to the segment of actual record shown in figure 4.

Then the mean lagged products, for lag interval $\Delta\lambda = p\Delta x$, are given by

$$C_{p,r} = \frac{1}{n - pr} \sum_{q=0}^{q=n-pr} H_q \cdot H_{q+pr}$$

for $r = 0, 1, \dots, m$ with $m \leq n/p$. (In all cases in this thesis p was equal to unity). Second, raw spectral density estimates are computed according to the formula

$$S_r = \Delta\lambda \left[C_0 + 2 \sum_{q=1}^{q=m-1} C_q \cos \frac{qr\pi}{m} + C_m \cos r\pi \right]$$

Third, refined spectral density estimates are computed according to

$$U_r = a_{i0} S_r + \sum_{j=1}^{\infty} a_{ij} [S_{r+j} + S_{r-j}]$$

The a_{ij} 's are the coefficients of the predetermined spectral window.¹ The criteria for the selection of an appropriate spectral window for a particular analysis are outlined in Blackman and Tukey; in the analysis used on the data for this thesis the coefficients for the spectral window chosen were $a_{30} = 0.50$ and $a_{31} = 0.25$ (all others being zero) hence we may write $U_r = 0.25 S_{r-1} + 0.50 S_r + 0.25 S_{r+1}$ and since only positive frequencies are considered the above estimates are doubled.

¹ A discussion of the spectral window employed in the analysis and its function can be found in appendix 1.

The above procedure is readily adaptable for a high-speed digital computer so that power spectra can be obtained with a minimum of computation and time on the part of the interpreter.

2.4 Power Spectra for Digitized Magnetic Data

A. The Data

The data used in this analysis were taken from several continuous strip chart recordings of the vertical component of the magnetic field as measured during the airborne magnetic surveys (1953-1960) of the Dominion Observatory.

The instrument used in making the measurements was a three-component airborne magnetometer designed and built at the Dominion Observatory (Serson, Mack, Whitham; 1957). It consisted of a field-measuring device mounted on a horizontal gyro-stabilized platform inside the aircraft. The accuracy with which the instrument measured the magnetic field at the magnetometer head was 0.1 degrees in declination and 10 gammas in the horizontal and vertical components. However, its accuracy in measuring the geomagnetic field was considerably lower because of uncertainties in the corrections applied to the readings for the magnetic field of the aircraft.

The error associated with one particular observation made in flight by an instrument of the type described

may be considered to be the sum of six independent errors which are listed below:

- (a) errors in measuring the magnetic field of the magnetometer with respect to the direction reference system,
- (b) errors in the direction reference system,
- (c) errors due to changes in the magnetic field of the aircraft,
- (d) errors due to magnetic disturbances,
- (e) errors in geographical position,
- (f) noise contributed by the measuring equipment.

Experimental results, obtained by flying on different headings over a region where the gradients of the magnetic field are small and accurate corrections for magnetic disturbances are available, showed that the probable error of a single observation, after correction for the magnetic field of the aircraft, was 40 gammas in the intensity of the horizontal magnetic vector and 10 gammas in the vertical component. The magnetic field of the aircraft changed over periods of a few weeks, producing an uncertainty in the corrections for the aircraft field which increased the probable error of survey observations to 60 gammas in the horizontal vector and 30 gammas in the vertical component. Errors in navigation and plotting the flight-lines and the effect of magnetic disturbances resulted in a further increase in the probable error of an observation, as plotted on the charts, to 100 gammas.

It was found by analysis of observations made in 1955 that the chief source of error was that of uncertainty in geographical position of the order of 4 miles which included errors in plotting the flight-lines as well as actual errors in navigation. The reason for such inaccuracy was the method of navigation which was by map reading, with 4 or 5 location estimates being made per hour. Aerial photography would have increased the accuracy but also increased the time required for plotting the results of the survey. Since the survey was conducted for the production of magnetic charts with a scale of 100 miles to the inch, the accuracy was considered sufficient. It should be pointed out also that over much of Canada radio aids to navigation such as Loran or Decca are not available; however, on oceanic flights the radio aid Shoran could be used for navigation.

The magnetometer head contained three orthogonal magnetic detectors of the saturated transformer type, which produced direct currents proportional to the fore-and-aft and transverse horizontal components and the vertical component. These and the heading of the aircraft were then fed into an analog computer which displayed continuously on strip chart recordings the declination in degrees, and the horizontal and vertical field components in gauss. An alternative display automatically presented the average values of these quantities over successive five minute intervals.

The chart speed was $3/4$ inch/minute and the average value of the field was taken every five minutes at 02 $1/2$, 07 $1/2$, 12 $1/2$, minutes. The centre of the averaging period was therefore at 00, 05, 10, minutes and the average value of the field during each five-minute interval was used as a base line for the next interval. The meter amplifier was zeroed at the end of each averaging period for about ten seconds and the location of the base line on the chart was found by observing the position of the trace during this time. This base line was always within ± 10 gammas of the zero of the chart.

The chart record gives the value of the field at the detector head and this, corrected for aircraft gives the estimated true value of the field measured at the aircraft's altitude. When the aircraft's magnetic heading and altitude are fairly constant the corrections to be applied remain the same. Care was taken in this work to avoid using those charts in which a large heading change was detected and with one exception the altitudes at which the field was recorded were all 7000 feet or greater.

The aircraft velocity was maintained at 180 miles/hour as closely as possible which meant that a 5 minute average corresponded to 15 miles of flight path and that one minute or $3/4$ inch of chart record was equivalent to 3 miles of flight path. Because of requirements due to digitizing,

the sampling interval was chosen to be 3 miles. The location of any particular anomaly in the oceanic profiles will no doubt be inaccurate by several miles; however, the features to be studied are of such a scale that their exact locations are not a binding factor.

Four charts for the continental region were chosen, one from the northern Canadian Shield area near the Arctic Circle and three more from the region between Ottawa and Edmonton. Four charts were also chosen for the oceanic regions, two from the Pacific and two from the Atlantic. The locations of the flight-lines for these charts are shown in figures 1, 2, and 3.

These recordings were then digitized with the sampling interval given above and the resulting data were plotted for each chart. In order to remove the trend in each profile which was largely due to the dipole field of the earth a base line¹ was drawn along the apparent trend through the data which had been joined together by a continuous curve. It can be seen from the resulting power spectra that this method satisfactorily removed the longer periods in the data due to the dipole field which was of no interest in this analysis. The plotted points were then revalued with respect to this new base line and the resulting data were then punched on cards and made ready for the power spectral analysis program for the digital computer.

¹ The base line shape was almost linear in the case of oceanic profiles, but it was more curvilinear for the continental profiles. See figure 4 for illustration.



FIGURE 1
for
flights 2,10,23,26



FIGURE 2
for
flights 19,20



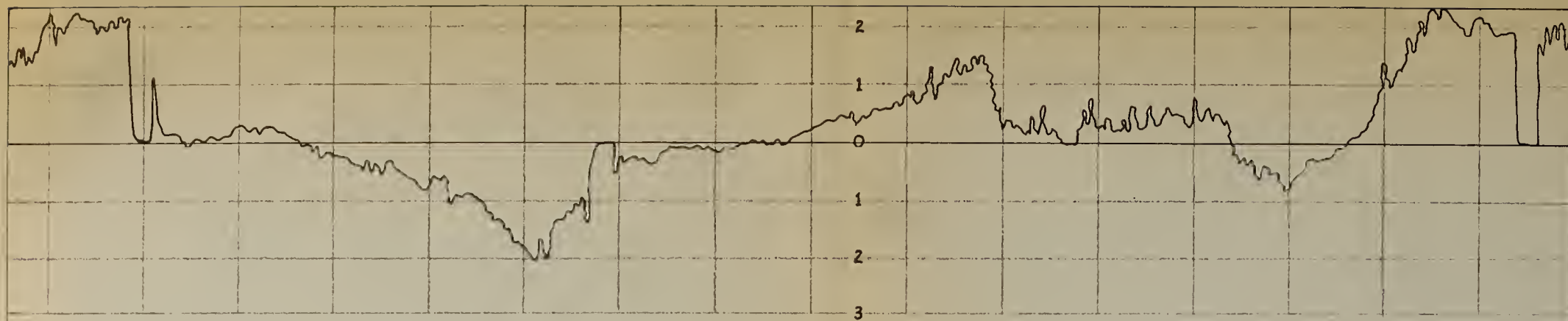


FIGURE 4a

Segment of Actual Record for Flight 23

Scale

Hor! 1"=4miles Vert! 1"=200gammas



FIGURE 4b

Curvilinear Dipole Trend for
Continental Profile (Fl. 2)

Scale

Hor! 1"=50miles Vert! 1"=600gammas

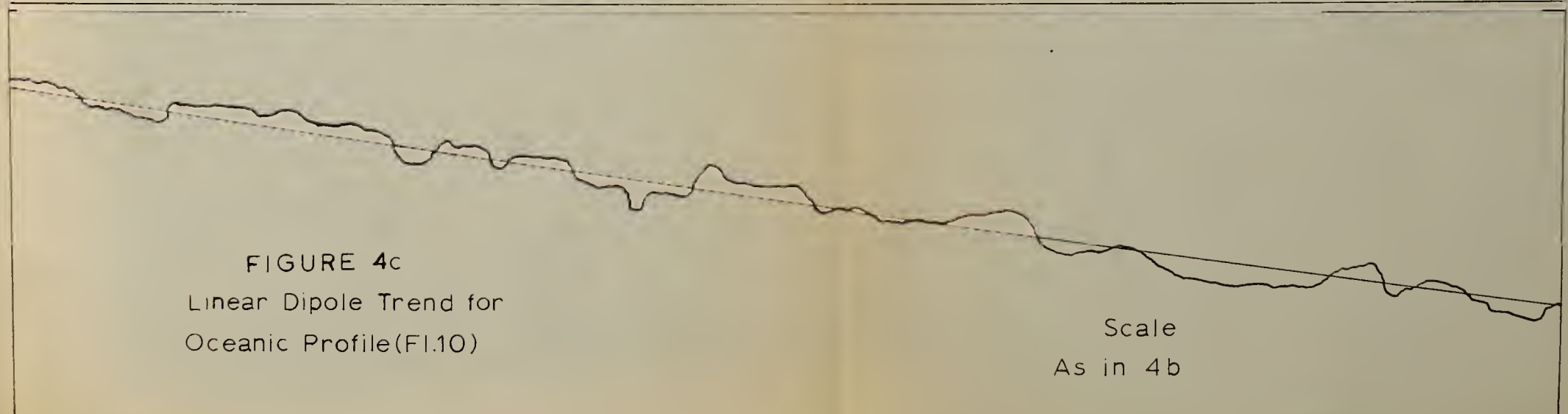


FIGURE 4c

Linear Dipole Trend for
Oceanic Profile (Fl. 10)

Scale

As in 4b

B. Reliability of Power Spectral Estimates

The reliability of the spectral estimates was investigated under the assumption that the true magnetic message was superimposed on Gaussian noise, that is, noise which exhibits a Gaussian distribution.

If $y_1, y_2, \dots, y_k$ are independently distributed according to a Gaussian distribution with a zero average and a unit variance then the quantity

$$\chi_k^2 = y_1^2 + y_2^2 + \dots + y_k^2$$

follows a chi-square distribution with k degrees of freedom. Now defining the coefficient of variation of χ_k^2 to be the ratio of the RMS deviation to the average value of χ_k^2 we obtain for its value $(2/k)^{1/2}$ where k is the number of degrees of freedom, RMS deviation $(\chi_k^2) = \sqrt{\text{var}(\chi_k^2)} = \sqrt{2k}$, and average $(\chi_k^2) = k$. Then as k increases, χ_k^2 becomes less variable and furthermore, the above relation holds for any multiple of χ_k^2 .

Now a convenient description for the stability or reliability of any positive estimate is its equivalent number of degrees of freedom. Since the above relation holds for any multiple of χ_k^2 , confidence intervals may be set up for the estimates based on the above consideration of the χ_k^2 distribution. For convenience in their manipulation in connection with power spectra these intervals are usually expressed in decibels.

For example, if an observed estimate of the true average power associated with a particular frequency band is given by z decibels, then provided it has 18 equivalent degrees of freedom, we can be 90% confident that the true average power lies within a 5 db interval symmetrical about the value z . As would be expected the smaller the decibel spread the greater the number of degrees of freedom required to assure that the true value lies within that particular interval. Thus the reliability of a particular observed estimate is functionally related to the number of equivalent degrees of freedom associated with that estimate. For a more detailed description of these decibel spreads and their derivation see Blackman and Tukey, pages 21 - 23.

The number of degrees of freedom will now be shown to be a function of the amount of data available such that the greater the length of record the more reliable the spectral estimates will be. It can be shown that for practical purposes the effective total length of record may be expressed by the approximation

$$X_n' \simeq X_n - \frac{p}{3} X_m \quad (\text{See Blackman and Tukey p.17})$$

where X_n' = effective length of record

X_n = total length of record

p = number of pieces of record.

X_m = maximum lag distance used.

The number of degrees of freedom is also expressible as the number of elementary frequency bands which, for design pur-

poses, may be approximated by

$$k \approx \frac{2X_n'}{X_m} \quad (\text{See Blackman and Tukey, p. 24})$$

In order to estimate the amount of data required for a known number of degrees of freedom one must consider the degree of resolution required in the power spectrum. As stated previously, the smoothing process for spectral estimates or the spectral window employed was that with coefficients 0.25, 0.50, 0.25, with all others being zero. This process supplies estimates every $1/2X_m$ cycles/mile where X_m is defined to be the maximum lag distance. Now adjacent spectral estimates will have windows which overlap each other considerably and hence they will not be completely resolved. However, consider the next-to-adjacent estimates, the overlap will be small and resolution can be said to be complete. Hence resolution may be defined as

$$\text{Resolution (in cycles/mile)} = 2 \frac{1}{2X_m} = \frac{1}{X_m}$$

In addition, as shown before, the Nyquist or folding frequency is given as

$$f_N = \frac{1}{2\Delta x} \quad \text{where } \Delta x \text{ is the sampling interval.}$$

Blackman and Tukey recommend that f_N be chosen such that

$$f_N = \frac{3}{2} f_{\max}$$

where $f_{\max}$ is the maximum frequency one wishes to analyze for

in the spectrum analysis.

It follows, therefore, that

$$\Delta x = \frac{1}{3f_{\max}}$$

and the necessary number of lags are therefore given by

$$m = \frac{X_m}{\Delta x} = 3 X_m f_{\max}.$$

The necessary number of data points then follow from

$$\begin{aligned} n &= \frac{X_n}{\Delta x} = (X'_n + \frac{p}{3} X_m) 3f_{\max} \\ &= 3 X'_n f_{\max} + p X_m f_{\max} \\ &= (3 \frac{X'_n}{X_m} + p) (X_m f_{\max}) \end{aligned}$$

$$\begin{aligned} \text{Now } X_m f_{\max} &= \frac{f_{\max}}{1/X_m} = \frac{\text{maximum frequency}}{\text{resolution}} \\ &= \text{number of resolved bands.} \end{aligned}$$

By substitution,

$$n = (1.5 k + p) (\text{number of resolved bands}).$$

In the spectral analysis carried out on the aeromagnetic data it was found that a resolution of 1/300 cycles/mile was required in order to define the peaks in the spectrum clearly. Furthermore, it was ascertained that the maximum frequency, $f_{\max}$, required would be of the order of 1/10 cycles /mile and appropriately the folding frequency was

chosen as $1/6$ cycles/mile which is approximately $3/2 f_{\max}$. The sampling interval defined by the selection of the folding frequency is given by

$$\Delta x = \frac{1}{2f_N} = 3 \text{ miles}$$

Then the number of lags required by the resolution is

$$m = \frac{X_m}{\Delta x} = \frac{300}{3} = 100$$

The number of resolved bands is given by

$$X_m f_{\max} = 300 \times 1/9 = 33.3$$

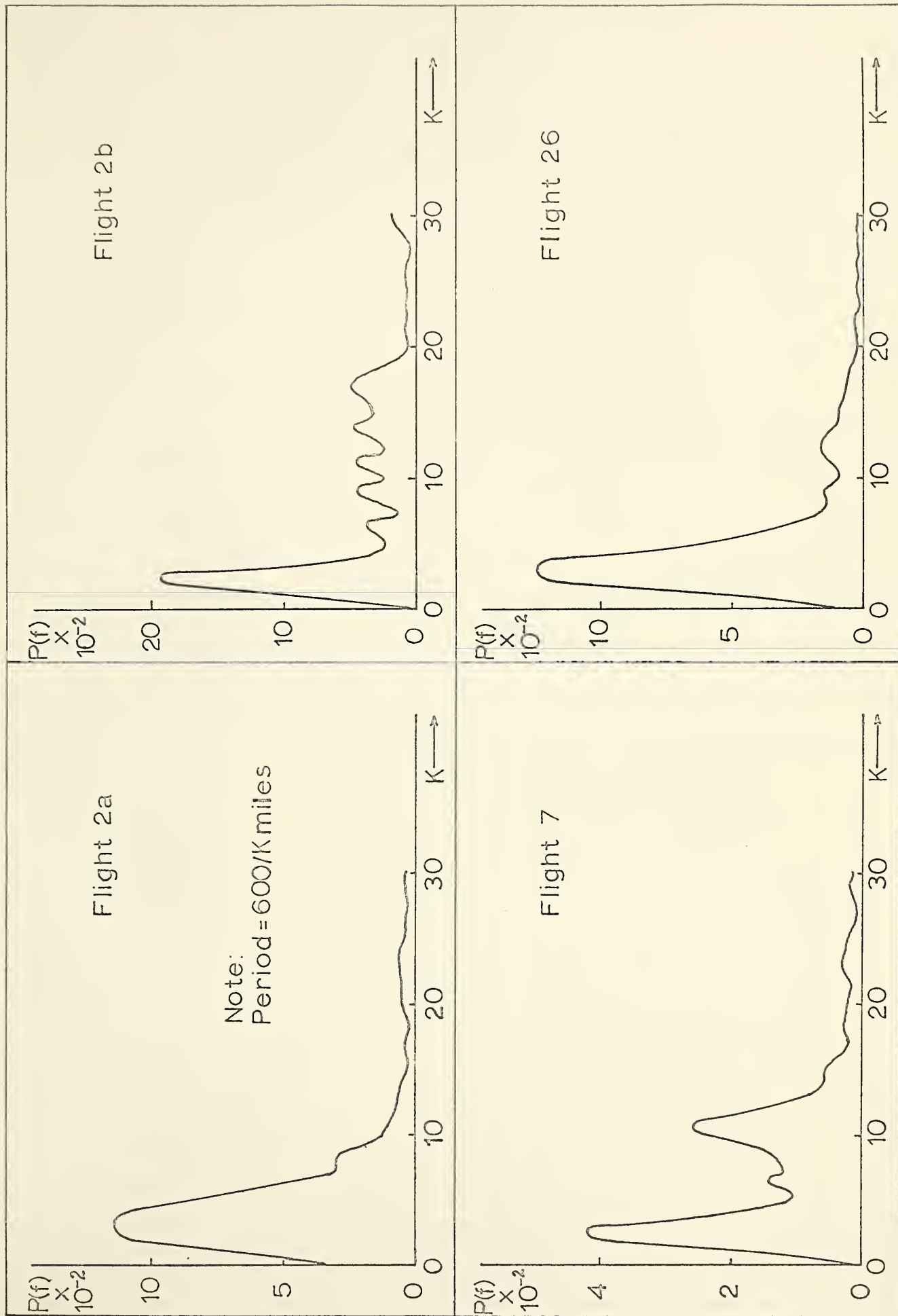
(since f_N was taken as $1/6$ cycles/mile,

$$\therefore f_{\max} = 2/3 \cdot 1/6 = 1/9 \simeq 1/10 \text{ cycles/mile})$$

As pointed out previously, the number of degrees of freedom required for a decibel spread of 5 db is 18 and there were 4 pieces of record for both continental and oceanic regions. Hence the total number of data points required on the 4 pieces of record is given by

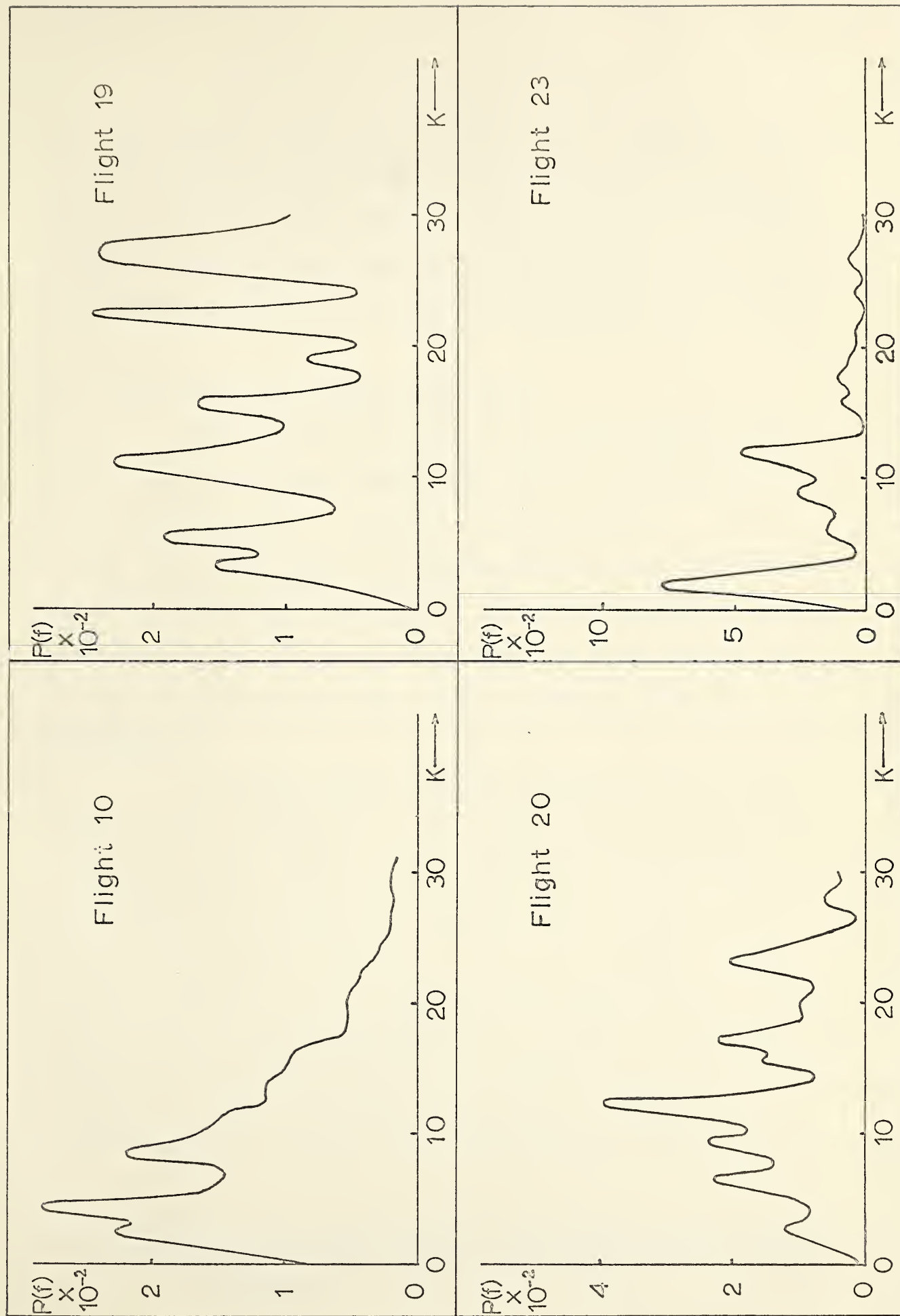
$$n = (1.5 \cdot 18 + 4) 33.3 \simeq 1000$$

The continental region spectrum was calculated using 1161 points and that for the oceanic region using 1257 points. The noise level on each spectrum is estimated to be about 50 units of average power or 70 db. The 90% confidence range about this level is 5 db which is symmetrical about



POWER SPECTRA FOR CONTINENTAL PROFILES

FIGURE 5



POWER SPECTRA FOR OCEANIC PROFILES

FIGURE 6

the 70 db level. That is, there is only a 5% probability that a peak in the spectrum, which is located above the 72.5 db level (53 power units), will be due to noise in the magnetic signal. Therefore any peak found above the 53 unit level can be taken as statistically significant with a 95% confidence.

C. Results of Power Spectral Analysis

The digitized data for each flight-line was processed using the power spectral analysis program¹ for the high-speed computer. A print-out was obtained for each flight-line which gave the power associated with each estimate at a particular frequency. Since the Nyquist frequency was taken to be $1/6$ cycles/mile then, as shown before, an estimate was obtained every $1/600$ cycles/mile from 0 to $1/6$ cycles/mile.

The power spectra for the aeromagnetic profiles flown over the continental and oceanic regions are shown in figures 5 and 6. The spectral estimates associated with these spectra are not statistically significant because of the insufficient record length in each case. However, it was assumed that each set of four profiles represented a random sample of the infinite population of profiles of the magnetic character of the continental and oceanic regions; thus, in order to obtain a statistically significant power spectrum

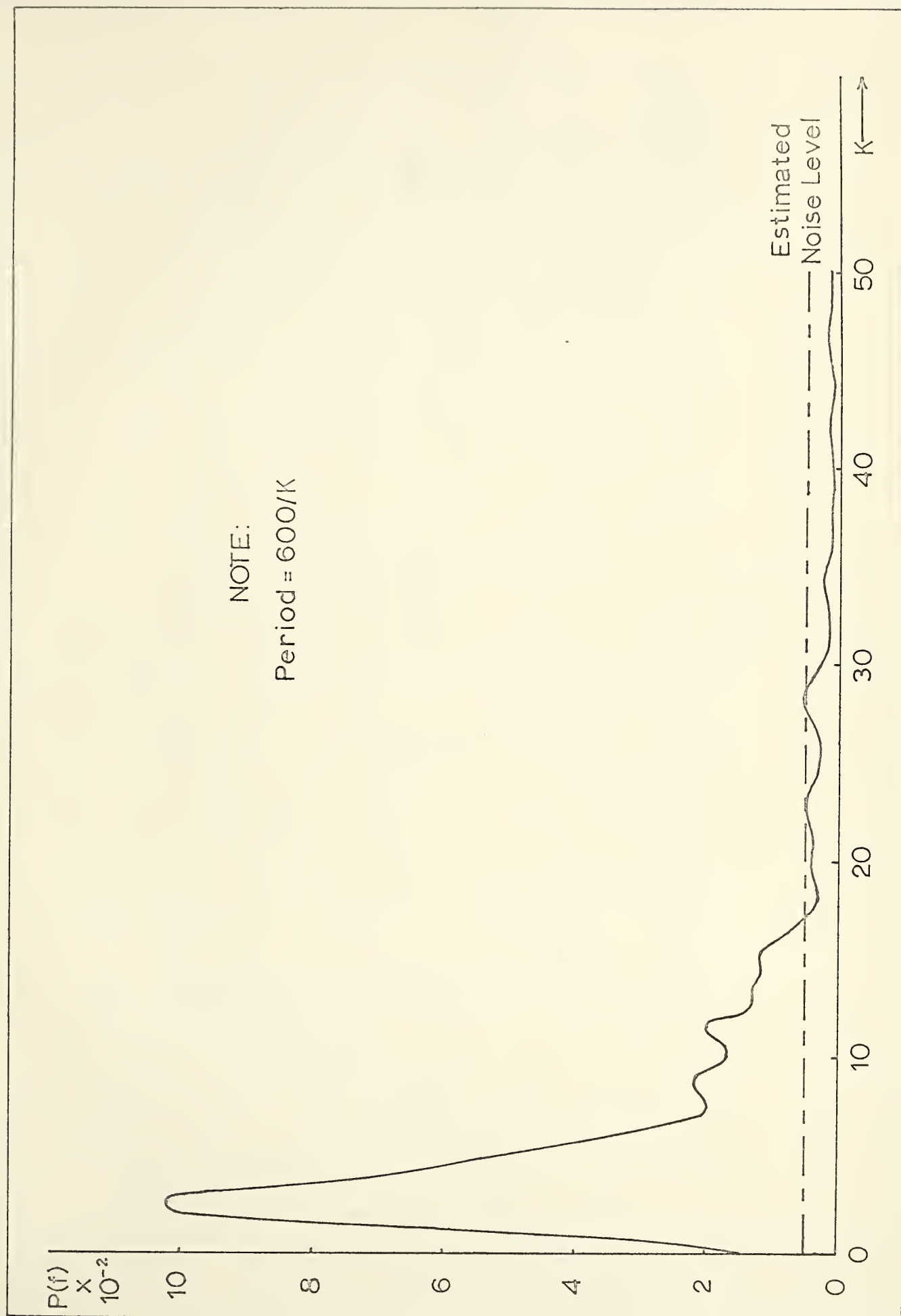
¹This program was written for the IBM 1620 digital computer by R. M. Ellis, Department of Physics, University of Alberta.

for each of the two regions the spectral estimates for each region (continental and oceanic) were weighted according to the number of data points in the particular length of record. The number of data points and corresponding weighting factors for each of the flight-lines involved are shown in table 1.

Table 1

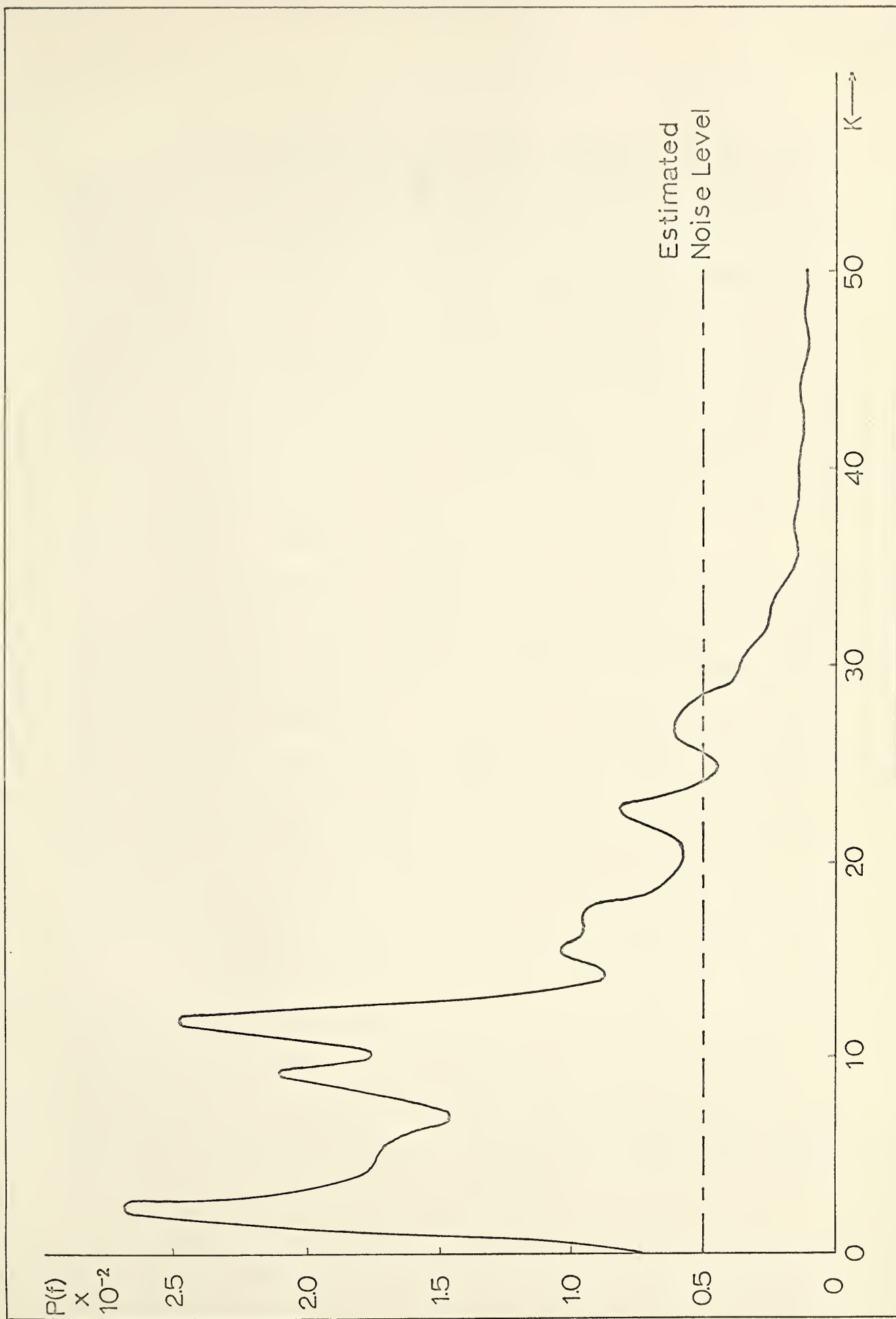
<u>Region</u>	<u>Flight No.</u>	<u>No. of Data Pts.</u>	<u>Weighting Factor</u>
Continental	2	180	0.1550
	2	378	0.3256
	7	369	0.3178
	26	<u>234</u>	<u>0.2016</u>
		1161	1.0000
Oceanic	10	612	0.4869
	19	159	0.1265
	20	270	0.2148
	23	<u>216</u>	<u>0.1718</u>
		1257	1.0000

The weighted estimates for the continental and oceanic regions were then plotted and the resulting spectra are shown in figures 7 and 8.



WEIGHTED CONTINENTAL POWER SPECTRUM

FIGURE 7



WEIGHTED OCEANIC POWER SPECTRUM

FIGURE 8

In interpreting these power spectra one must not assume that, because the spectra contain certain dominant frequencies, the magnetic character of the crust is necessarily periodic. Such a conclusion is unwarranted because of observations made of surface geology; however, there are certain marked similarities and dissimilarities between the spectra for the two regions which should be investigated. A detailed discussion of these features will be given in chapter 4; at this time we wish only to outline the reasons for the application of numerical filtering to the data.

As mentioned in section 2.1, it was a general belief that anomalies of high intensity, due to near-surface magnetic structures, might mask certain long period anomalies with relatively low amplitudes. In order to reveal these long period features more clearly and accurately than interpolation by the human eye could accomplish, it was decided that the data should be convoluted with appropriate discrete filters or weighting functions. This type of smoothing process could also be applied more quickly than that of smoothing by hand.

Although the peaks on both spectra appear to occur at approximately the same frequency in all cases, we shall investigate fully only one significant, common peak, that which occurs at a period of approximately 250 miles in both spectra. Since the trend due to the dipole field was removed,

this long period is evidently due to some broad, deep structure which seems to be common to both continental and oceanic magnetic crusts. One such structure which might cause such broad anomalies could be an upwarping or downwarping of the Curie point geotherm. This possibility will therefore be investigated in the next chapter with the aid of numerical filtering.

III. NUMERICAL FILTERING

3.1 Introduction

The procedure of applying a weighting function or filter to a set of data in order to eliminate undesirable noise from the signal is nothing more than a type of smoothing process. Just as the formation of the autocovariance function is the smoothing of the signal by the signal itself, the action of a filter (a low-pass or bandpass filter) is to remove by smoothing undesirable high frequency components.

In numerical filtering of equi-spaced data the filter form is that of a weighting function which is composed of several discrete points separated from one another by the sampling interval for the data in question. The shape of this weighting function is governed by the frequency characteristics which the filter must have in order to perform the required operation.

The proper application of a filter to any type of data inevitably requires some predetermined information about the message to be extracted from the data. In geophysics, such a priori knowledge might be supplied through the geology or as in this case, by a power spectral analysis, provided the input signal is assumed to be random. Such an analysis defines the range of frequencies against which the filter must discriminate.

In general, there are three basic types of filters: low-pass, bandpass, and high-pass. The low-pass discriminates against high frequencies, the bandpass against both low and high frequencies, and the high-pass against low frequencies. In most geological problems the filter required will either be of the low-pass or bandpass type since most of the noise in the signal is usually caused by near-surface geologic effects which are composed mainly of high frequency components.

One distinct advantage of numerical over electrical filters is that the weighting function to be applied to the data does not need to be physically realizable. That is, one does not have to reproduce the weighting function as the output of an electrical circuit. Once the frequency characteristics of the weighting function have been determined to be those required to perform the necessary filtering operation the filter is thus designed. In this connection, it is best to point out that linear filters behave well in the neighbourhood of the signal origin. They are, in fact, independent of signal origin; but, it is best to choose an origin which is at least one weighting function away from the approximate location of the anomaly being sought in order that part of the desired message not be lost in filtering.

3.2 Filter Design

In designing a filter or weighting function there are several criteria to which one must adhere in order that the filter perform the desired operation reliably. These criteria in order of importance are:

- (1) the filter must preserve the message frequency while discriminating against undesirable high frequency noise.
- (2) the filter must not distort the true shape and amplitude of the message.
- (3) there should be no phase shift due to filter application, if at all possible.
- (4) the discrete weighting function factors should be normalized with respect to unity in order that unit response be given to a function of infinite period.

In order to determine if a filter satisfies the first criterium one must study the frequency characteristics of the weighting function. This is usually accomplished by computing the Fourier transform of the weighting function which, in effect, gives the response of the filter to a given input with respect to the frequency of that input. In this connection, it is best to point out that one can proceed from the Fourier transform to filter shape as Ormsby describes (1960), that is, one can design the desired transform function and then derive the corresponding weighting

function, or one can experiment with filter shapes directly (Goldstein, 1962). The latter method is quite satisfactory for low-pass filters; in this method one must determine the frequency characteristics of the filter each time the weighting function shape is changed.

It was found that this procedure was adequate if one made the weighting function the same width as the ~~width~~ (in miles) of the message sought. This period was taken from the power spectrum for the particular flight profile to which the filter was to be applied. In most cases this period never varied more than 20 to 30 miles on either side of the average long period of 250 miles appearing on both continental and oceanic power spectra. The shape of the weighting function was then altered until the Power Frequency Transfer Function¹ satisfied the specified frequency characteristics.

The definition of the Transfer Function is derived from the following mathematical analysis of the filtering process.

The input to a linear filter is related to its output by the following integral equation known as the convolution integral:

$$\xi_0(x) = \int_{-\infty}^{\infty} \xi_1(x-\delta) W(x) dx \quad \dots(1)$$

where x = space variable

W = weighting function

δ = lag

¹Power Frequency Transfer Function - the absolute value of the Fourier transform of the weighting function, all squared.

To express a function of a time or space variable as a function of frequency one need only take the Fourier transform of the function. For example,

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi(x) e^{+i\omega x} dx \quad \dots(2)$$

and inversion gives,

$$\xi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(\omega) e^{-i\omega x} d\omega$$

where $\omega/2\pi$ is the frequency in cycles/mile.

The Fourier transform of the integral equation (1) gives a simple input-output relation in terms of the frequency variable, ω .

$$E_o(\omega) = E_i(\omega) Y(\omega) \quad \dots(3)$$

where the functions $E_o(\omega)$, $E_i(\omega)$, $Y(\omega)$ are the Fourier transforms of the functions $\xi_o(x)$, $\xi_i(x)$, $W(x)$ respectively.

From (3) follows the definition of the Power Frequency Transfer Function,

$$|Y(\omega)|^2 = \left(\frac{E_o}{E_i}\right)^2 \quad \dots(4)$$

The analogy between the above expression and that of the ratio of output over input voltage in an electrical circuit is obvious. Hence the name Power Frequency Transfer Function.

Now

$$Y(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(x) [\cos(\omega x) + j \sin(\omega x)] dx \quad \dots(5)$$

Since the weighting function is given by a series of discrete points separated from one another by the sampling interval Δx , then,

$$Y(\omega) = \frac{\Delta x}{\sqrt{2\pi}} \sum_{x_1=-\infty}^{\infty} W(x_1) [\cos(\omega x_1) + j \sin(\omega x_1)] \dots (6)$$

For a symmetrical weighting function,

$$Y(\omega) = \sqrt{\frac{2}{\pi}} \Delta x \sum_{x_1=0}^{\infty} W(x_1) [\cos(\omega x_1) + j \sin(\omega x_1)] \dots (7)$$

Let
$$Y(\omega) = \sqrt{\frac{2}{\pi}} \Delta x (A + jB)$$

$$\therefore |Y(\omega)| = \sqrt{\frac{2}{\pi}} \Delta x |\sqrt{A^2 + B^2}| \dots (8)$$

and the Power Frequency Transfer Function is given by,

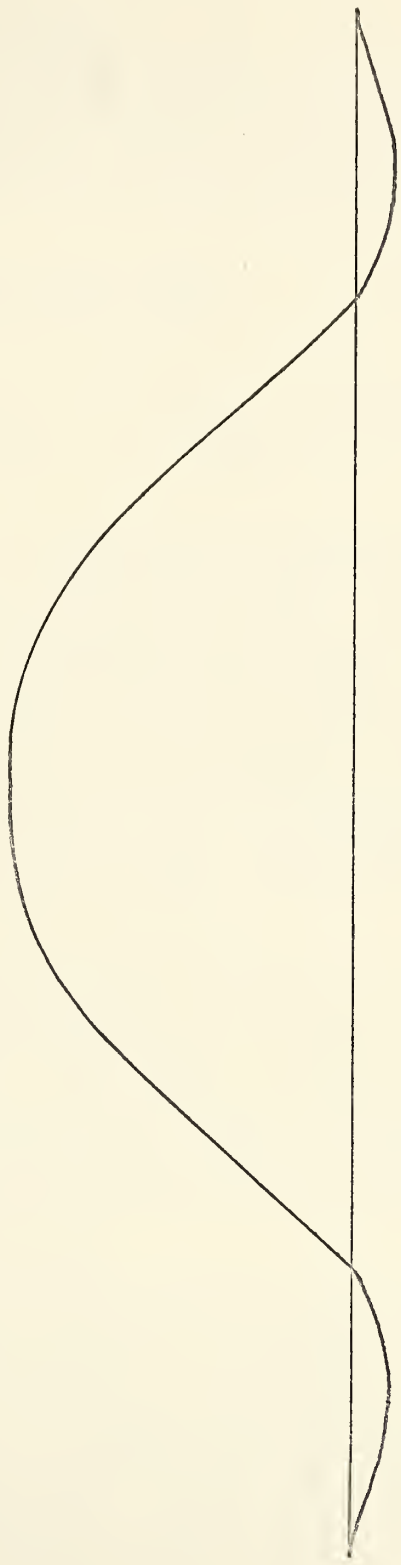
$$|Y(\omega)|^2 = \frac{2(\Delta x)^2}{\pi} (A^2 + B^2) \dots (9)$$

where
$$A = \sum_{x_1=0}^{\infty} W(x_1) \cos(\omega x_1)$$

$$B = \sum_{x_1=0}^{\infty} W(x_1) \sin(\omega x_1)$$

and
$$\sum_{i=1}^N W(x_i) = 1$$
 for a weighting function of length $N\Delta x$.

The third of the aforementioned criteria is satisfied if the weighting function is symmetrical with respect to its centre point. This construction also simplifies the summation



Filter 7

Scale:

Horl 1"=15miles

Vert 1"= 200 gammas

given in equation (7) above.

A more convenient form of the Transfer Function for plotting is obtained if one converts the quantity to decibel units.

From equation (9) then,

$$|Y(\omega)|_{\text{db}}^2 = 10 \log_{10} |Y(\omega)|^2 \quad \dots(10)$$

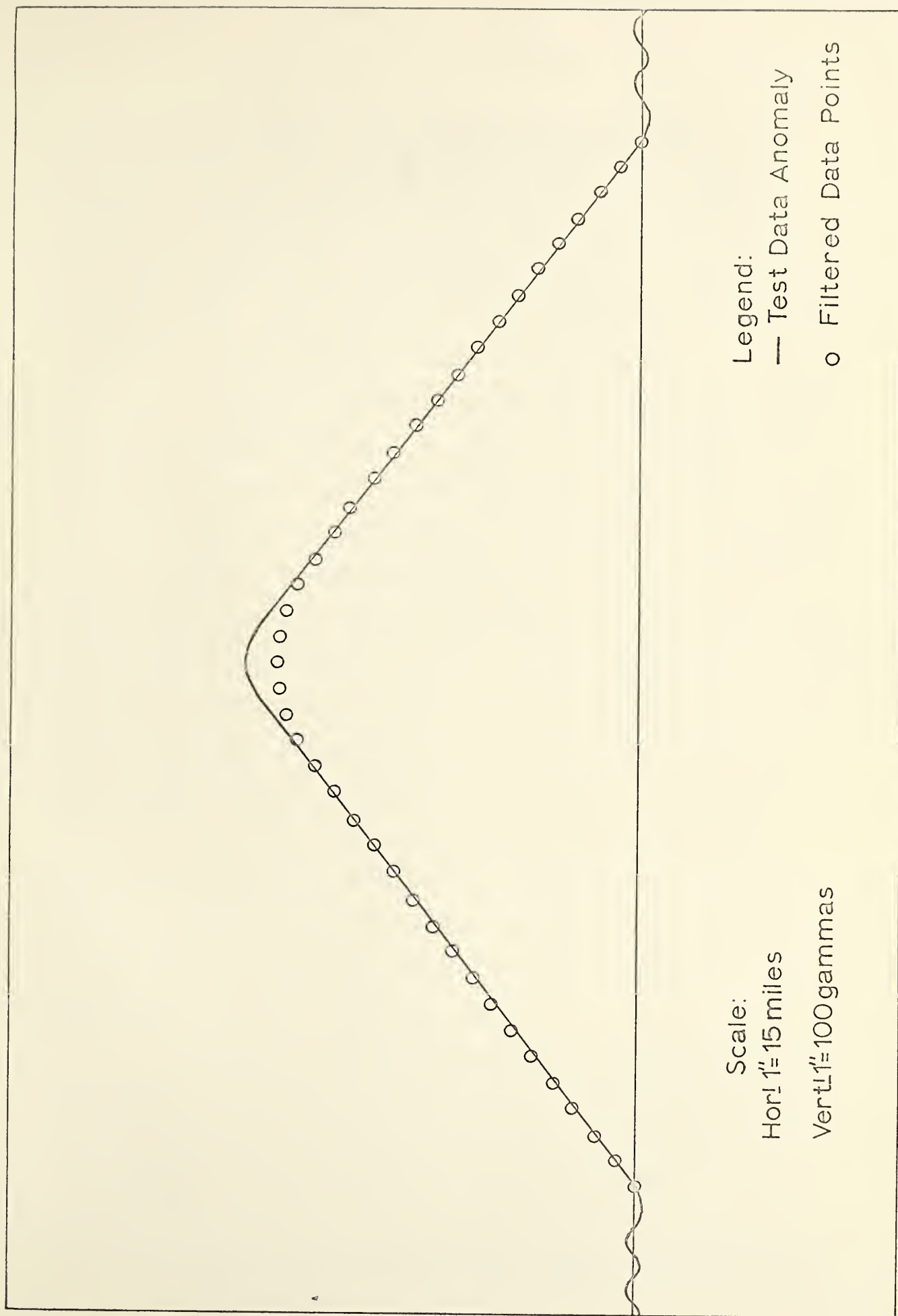
It should be mentioned that the summations in equation (9), though taken for x_i from 0 to ∞ really only apply, effectively, over the finite length of the weighting function since beyond the end points the function is zero valued. These calculations can be easily programmed for the digital computer and one can then obtain the Transfer Function quite rapidly.

One of the weighting functions used in the numerical filtering of the flight data is shown in figure 9. This filter was obtained after seven refinements of the original trial filter and satisfies not only the frequency requirements but also criterium 2.

The Transfer Function for this filter is shown in figure 10. It can be seen that the filter is reasonably sharp and therefore will discriminate against high frequency components quite adequately. The very low frequency components are not a factor here since, as can be seen from the



POWER FREQUENCY TRANSFER FUNCTION FOR FILTER 7



TEST FOR DISTORTION DUE TO FILTERING

FIGURE 11

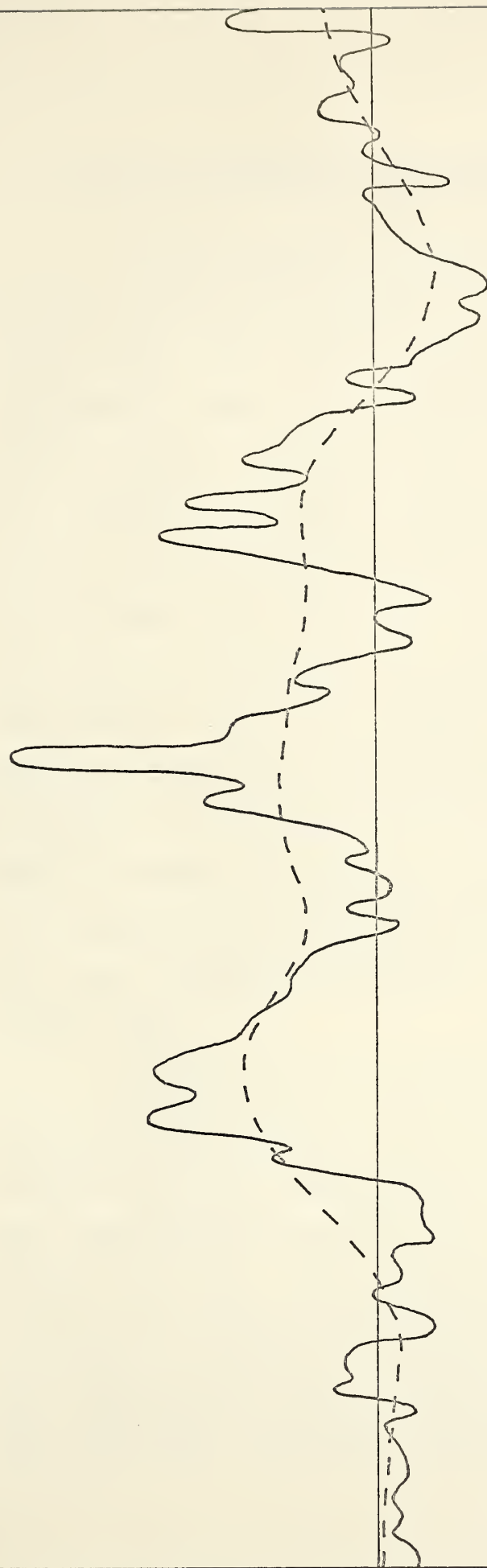
power spectra, they have been adequately removed. In the frequency range of interest, the filter is very slightly suppressive; however, it is very close to the zero decibel axis and therefore will have approximately a 1:1 ratio for response/input. This factor is very important in order that criterium 2 be satisfied as closely as possible.

In addition to this factor, the weighting function must be so constructed that the shape of the message is not appreciably distorted. This is necessary since depth calculations for magnetic anomalies are very dependent on the shape of the anomaly. In order to check the filter out on this latter stipulation an anomaly was constructed with test data whose length was identical to that of the weighting function. This test data was then convoluted with the filter to determine the amount of distortion due to filtering. The test data anomaly and the filtered result are shown in figure 11.

The filter thus duplicates the anomaly slopes quite accurately and the amplitude suppression is only 25 gammas in 300 or 8 per cent. The filter can also be suitably altered in shape and length to handle messages with periods in the 200 to 300 mile range.

Having succeeded in designing an appropriate filter the various flight profiles were then convoluted¹ with this filter (for various periods) producing a filtered profile. An example of such a profile is given in figure 12.

¹The mathematical procedure for this convolution which was programmed for the digital computer is outlined in appendix 2.



Legend:
— Observed
Profile
--- Filtered
Profile

A Portion of the Convolution
of Flight 7 Data with Filter 7

Scale:
Hor. 1" = 30 miles
Vert. 1" = 200 gammas

3.3 Results of Numerical Filtering

As mentioned in section 3.2, the filter produces its best accuracy when applied to a signal containing an anomaly whose width is identical to that of the filter itself. Consequently, when the filtered data were obtained from the digital computer they were then scanned for anomalous features whose widths were within ± 3 miles of that of the filter applied. It was found that an anomaly, whose width was greater than or less than that of the filter applied by more than 3 miles, was not shortened or lengthened as a result of filter application. That is the width of the anomaly was relatively constant under filter application as long as the filter width was within 10 - 20 miles of that of the anomaly. Therefore, when an anomaly of greater or lesser width than that of the filter applied was found the appropriate filter of proper width was then applied insuring a minimum of distortion of anomaly shape.

Because of the relatively large periods of the anomalies being sought it was difficult to find more than one on any particular flight profile. Only on flights 2 and 10 were two anomalies found in each case; flights 7, 20, and 23 produced one each. The anomalies produced by filtering profiles for flights 17 (Atlantic ocean) and 26 (continental) were comparatively small in width (having a period much less than 200 miles) and so did not qualify for interpretation.

In general, it appears, from the seven anomalies analyzed, that the oceanic anomalies are relatively smaller in amplitude than those of the continental profiles. It should also be pointed out that all of the oceanic anomalies were positive while those of the continental region were all negative. It is suspected that this latter feature may not be significant in view of the very small number of anomalies found on the profiles. A general description of the anomalies found is given in table 2.

<u>Table 2</u>				
<u>Region</u>	<u>Flight No.</u>	<u>A n o m a l y F e a t u r e s</u>		
		<u>No.</u>	<u>Anomaly width (miles)</u>	<u>Max. Amplitude (gammas)</u>
Oceanic	10	A	117	44
		B	102	107
	20	A	108	67
	23	A	144	228
Continental	2	A	100	188
		B	121	348
	7	A	126	204

It should be noted that anomaly width as referred to in table 2 is that portion of the anomaly which occurs between the zero amplitude crossings near each end of the anomaly.

A more complete discussion of these anomalies and their interpretation will be given in chapter 4. Diagrams of the anomalies will be given there also.

IV. INTERPRETATION OF RESULTS

4.1 Interpretation of Power Spectra

An interpretation of the contrasting and the similar features of the weighted power spectra shown in figures 7 and 8 in chapter 2 will now be given.

A power spectrum for continuous or discrete data displays estimates of the true average power associated with particular frequency band widths over a specified range of frequencies. The amplitude of any estimate is a function of the frequency of occurrence of the corresponding period as well as the energy associated with this period.

Through observation of the eight power spectra obtained (see figures 5 and 6, chapter 2), the maximum noise level was approximated as 50 db, due to the gradual decline from this level on all of the spectra in the shorter period range (< 30 miles). Thus the four peaks which appear on each of the weighted spectra can be said to be statistically significant in view of the analysis of reliability of spectral estimates given in section 2.4. That is, one can be 95 percent confident that these peaks are not due to some anomalous feature of the noise spectrum.

Bearing these facts in mind, let us analyze the frequency characteristics of the continental and oceanic spectra shown in figures 7 and 8. In both cases there is a

sharp rise from the zero frequency value to the peak at the frequency of approximately $1/250$ cycles/mile. The large gap between these two frequencies is prominent in both spectra and shows that there is only a comparatively small amount of energy associated with those periods which lie between infinity and 250 miles. Thus it appears that the removal of the dipole field trend causes those long periods, supposedly associated with the core-mantle boundary, to disappear. This result seems to agree with that of Alldredge and Van Voorhis (1961). Recall from chapter 1 that, using a simple type of harmonic analysis, they attributed those features of extremely long period to the core-mantle boundary sources and those whose crossover distances lay between zero and 260 nautical miles were said to be due to crustal magnetic materials. It appears that the sources which produced the four anomalous peaks on each of the power spectra can be classed as crustal or near-crustal structures.

Although these spectra are only two samples from the infinite population of possible spectra for crustal magnetic character, there could be some significance to the fact that the location of each of the four peaks on the continental spectrum is approximately coincident with that of the corresponding peak on the oceanic spectrum. The four periods under consideration are in the neighbourhood of 250, 65, 50, and 40 miles. These periods correspond to

anomaly widths of roughly 125, 33, 25, and 20 miles respectively, since an anomaly for a relatively broad structure has a shape which approximates half a sine wave.

One possible explanation is that each of these periods is characteristic of structures which occur at a certain depth; that is, the higher frequency anomalies are due to near-surface narrow magnetic structures and as the depth increases the width of the source must increase in order to compensate for attenuation of the signal sent out by the source. The attenuation of the signal from a magnetic source below the surface is proportional to both depth and frequency (Dean, 1958). Hence the signal from a narrow structure (higher frequency) would become gradually negligible if the depth to this structure were increased. Also, in most areas, there is a rapid lateral change in susceptibility contrast among the various rock types on or near the surface. These changing contrasts, though small, can give rise to narrow anomalies and broad anomalies due to near-surface structure are not detectable. Due to attenuation these sharp lateral changes gradually become smoothed over and in order to produce an anomaly in the earth's field measured at the surface a greater susceptibility contrast is required as the depth increases. This, in turn, requires a greater amount of lateral distance along the structure in order that a sufficient contrast is achieved, thus giving rise to broader

anomalies. Therefore, one might say that the period of a magnetic anomaly increases with the depth to the structure in which the source is located.

This increase in anomaly period will continue until the Curie point geotherm is reached. At this depth there will be a sharp drop in susceptibility and the rocks will essentially cease to be ferromagnetic any longer. It is therefore hypothesized that this sharp drop in susceptibility corresponds to the sharp drop from the 250 mile peak to the zero frequency value on each of the power spectra; that is, the 250 mile period anomalies are due to structures associated with the Curie point geotherm.

Having analyzed the frequency characteristics, let us examine the amplitudes of the peaks occurring on both spectra. As stated previously, in more general terms, the amount of power associated with any one particular spectral estimate is dependent on the frequency of occurrence of the corresponding period and on the amount of magnetic intensity associated with the structure which gives rise to an anomaly with such a period. It is evident that the three smaller peaks on both spectra occur in roughly the same range of amplitude while that of the 250 mile period is much larger for the continental region than that of the oceanic region. The frequency of occurrence of the smaller structures associated with the smaller periods will no doubt be much higher

than that of the structures with the 250 mile period. Because of this high rate of occurrence any difference in magnetic intensity could be counter-balanced by a slight difference in frequency of occurrence between continental and oceanic regions. However, this should not be the case for the longer period anomalies. The investigation of these long periods by numerical filtering showed that the frequency of occurrence for both regions was roughly equal but that the amplitudes associated with the broad anomalies was generally less for the oceanic region. Although these amplitudes are not completely accurate, the trend is still evident and points to the fact that if these broad anomalies occur at approximately the same depth in both continental and oceanic crusts then the intensity of magnetization is greater for the continental structures than for the oceanic ones. This hypothesis is supported by the findings of Serson and Hannaford (1957).

4.2 Interpretation of Numerically Filtered Anomalies

In section 3.3 a brief description of the seven long period anomalies revealed by numerical filtering was given. In this section a more complete description of the anomalies will be given and it will also be shown that the sources of these anomalies can be interpreted as anomalous Curie point geotherm structures.

First of all, one must realize that, although the long period trends (250 miles) are quite prominent on the power spectra for both regions, numerical filtering cannot reveal these broad anomalies with the same clarity as a power spectrum might detect their presence. This is due to the fact that the superposition of higher frequency near-surface features on the broad anomalies can distort them beyond recognition by a filtering technique. Hence only in those areas where the magnetic character of shallower structures is relatively homogeneous can the broad anomalies be extracted in their entirety by filtering. This, coupled with the fact that the anomaly width was large compared to the flight lengths available, accounts for the small number of these broad anomalies which were available for interpretation after numerical filtering was applied.

In geophysics, for any potential field anomaly measured at the surface there are basically two methods by which an interpretation for the possible structure causing the anomaly may be made. The word possible is employed here since it is a well-known fact that potential field anomalies can have several solutions, all equally feasible.

The first of these methods is that which employs downward or upward continuation. The basic concept of this method is that one calculates a set of coefficients which are then convoluted with the anomaly data such that the

anomaly is continued downward to the depth at which the source is located or an anomaly can be calculated for a theoretical structure at a certain depth and then continued upward. In the former method the calculated anomaly will begin to oscillate once the approximate location of the true depth to the source is passed. In the latter method the anomaly continued upward to the surface must match the observed anomaly, then the theoretical structure is said to be a possible source for such an anomaly. In downward continuation one can project either the potential or the field downwards, the former provides a better insight as to the shape of the source. There are several treatises on this method of continuation, the reader is referred to Peters (1949) and to Dean (1958) for a more comprehensive account of the application of this method.

The second method is that of geometric construction of a cross section for which the theoretical anomaly is calculated. The shape of the cross section as well as its depth are then altered until the resulting theoretical anomaly approximates as accurately as possible the shape of the observed anomaly.

It was first thought that the interpretation of the seven anomalies found by numerical filtering might best be accomplished by calculating the potential at the surface

Magnetic
Field
Surface

Magnetic
Material

---Geotherm

Non-Magnetic
Material

Negative anomaly due to thinning
of magnetic material

Figure 13a

Magnetic
Field
Surface

Magnetic
Material

---Geotherm

Non-Magnetic
Material

Positive anomaly due to thickening
of magnetic material

Figure 13b

using the filtered vertical component of the field and downwardly continuing this potential field to the true depth. However, this idea was abandoned when it was found, as pointed out by Dean (1958), that the data errors are amplified by a factor of approximately twice the difference of the maximum adjacent coefficients. The maximum adjacent coefficients for the depths to which the anomalies were to be projected in this analysis differed by as much as a factor of 10^5 in some cases. For this reason the anomalies were fitted geometrically according to the second method outlined above.

It was hypothesized that the anomalies were due to an upwarping or downwarping of the Curie point geotherm. As shown in figure 13, an upwarping of the geotherm gives rise to a negative anomaly in the earth's field as measured at the surface and a downwarping causes a positive anomaly to appear at the surface if the susceptibility contrast is sufficient in both cases.

The seven anomalies were then fitted geometrically with theoretical anomalies using the above facts and the well-known theory¹ for the calculation of the vertical magnetic attraction of a distribution of magnetic charge induced by the earth's field on the surface of a two-dimensional structure. The following assumptions and estimates were made in employing this theory:

¹This theory, as outlined by Press and Ewing (1952), is developed fully in appendix 3 for the two types of theoretical structures used in this interpretation.

- (1) it was assumed that the polarization induced by the earth's field on the body was in the vertical direction only since the majority of the anomalies were relatively symmetrical.
- (ii) a simplified cross section symmetrical about its central axis was assumed for each structure, again because of the symmetry of the observed anomalies.
- (iii) the maximum thickness of each structure was assumed to be small with respect to its width (3 miles as compared to 100 miles in most cases) in order that the hypothesis of the source being a warping of the geotherm be plausible.
- (iv) the interpreter must estimate the lateral extent of each anomaly and also take into consideration the effects of adjacent anomalies in order that the "end" errors be minimized.

In figures 14, 15, 16, 17, 18, 19, and 20 are shown the final two dimensional cross sections together with the corresponding theoretical and observed anomalies. These interpretations are by no means unique solutions for the observed anomalies; but they are thought to be the best trial-and-error solutions under the assumptions made by the interpreter.

In order to investigate the possibility that the susceptibilities of continental rocks were, in general, greater than those of oceanic rocks as suggested by the interpretation of the power spectra, the susceptibility contrasts

were calculated using the maximum value for the vertical intensity for each of the observed and theoretical anomalies.

The values for the vertical component of the earth's field were taken from the aeromagnetic map of the vertical component of the earth's field as published by the Hydrographic Office of the United States Navy (1955). Since the base line, drawn through each of the original profiles to remove the dipole field, was not absolutely accurate these values for susceptibility contrast can only be taken as a rough estimate of the true value for the particular structure in question. The susceptibility contrast estimates together with the depth below the surface of the earth to the theoretical structures are summarized in table 3.

Table 3

Region	Flight No.	Earth's Vertical Component (oersteds)	No.	Anomaly	Features
				Depth ¹ (miles)	Susceptibility Contrast (c.g.s.) x 10 ³
Oceanic	10	0.42	A	21.5	5
		0.38	B	19.5	10
	20	0.48	A	18.5	6
	23	0.44	A	5.0	15
Continental	2	0.58	A	22.5	15
		0.58	B	25.5	34
	7	0.59	A	29.6	15

¹The depths for continental features are referred to the land surface while those for oceanic features are referred to the ocean floor. The ocean depths are only approximate and were obtained from the "Advanced Atlas of Modern Geography", (1962).

The calculations of susceptibility contrast, as summarized in table 3, seem to support the idea of higher susceptibilities for continental rocks as suggested by the interpretation of the power spectra in section 4.1. In general, the depths to the anomalous structures, which have been hypothesized to be Curie point geotherm structures are slightly larger for continental features.

The anomaly for flight 23 does not agree with the pattern set forth by the other six anomalies; this could be due to the fact that the anomaly location is quite close to the terminal point of the flight, Shemya in the Aleutian Islands. This is a region of high volcanic activity and the temperature gradient here may be much higher than the mean gradient for the oceanic crust, thus giving rise to a shallow Curie point geotherm. The anomaly depth is also within range of the Mohorovicic discontinuity for the oceanic crust (approximately 4 miles) and a warping of the discontinuity might be the cause.

Generally, it appears that the depths to the oceanic sources are well below the average depth to the Mohorovicic discontinuity, whereas, those of the continental features

seem to be in the discontinuity depth range for the continental crust (approximately 25 miles). Therefore, it is conceivable that the Curie point geotherm is quite closely associated with the discontinuity for the continental crust; or, perhaps some or all of these continental anomalies are due to anomalous magnetic features of the discontinuity itself. The latter interpretation then implies that the geotherm depth would be deeper still and perhaps the susceptibility contrast there is insufficient to produce an anomaly on the earth's field at the surface.

The accuracies of the depths and susceptibility estimates summarized in table 3 are difficult to estimate. However, it is safe to say that the accuracy of the depth estimates is better than that of the susceptibility contrasts. In view of the method of interpretation and the method of removing the long period dipole field trend from the original data, the depth estimates are probably accurate to within ± 3 miles and the susceptibility contrasts to within ± 50 per cent of the values shown in table 3.

Locations of Anomalies in Figures 1,2,3

0 gammas

Flight Level
Alt.=8000'

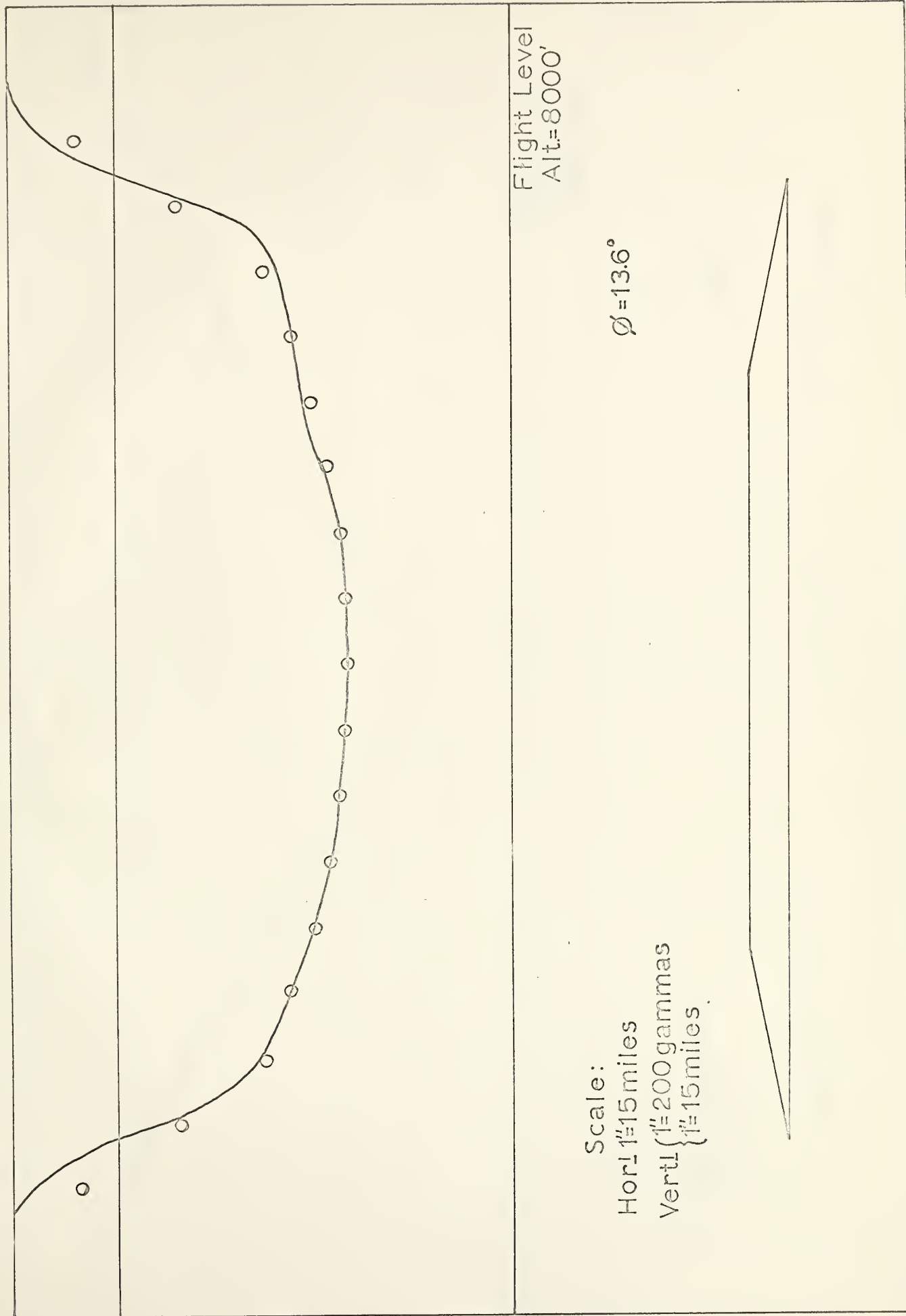
- Legend:
- Observed Anomaly
 - o Theoretical Anomaly

Scale:
Horl 1"=10 miles
Vert 1"=100 gammas (above)
1"=10 miles

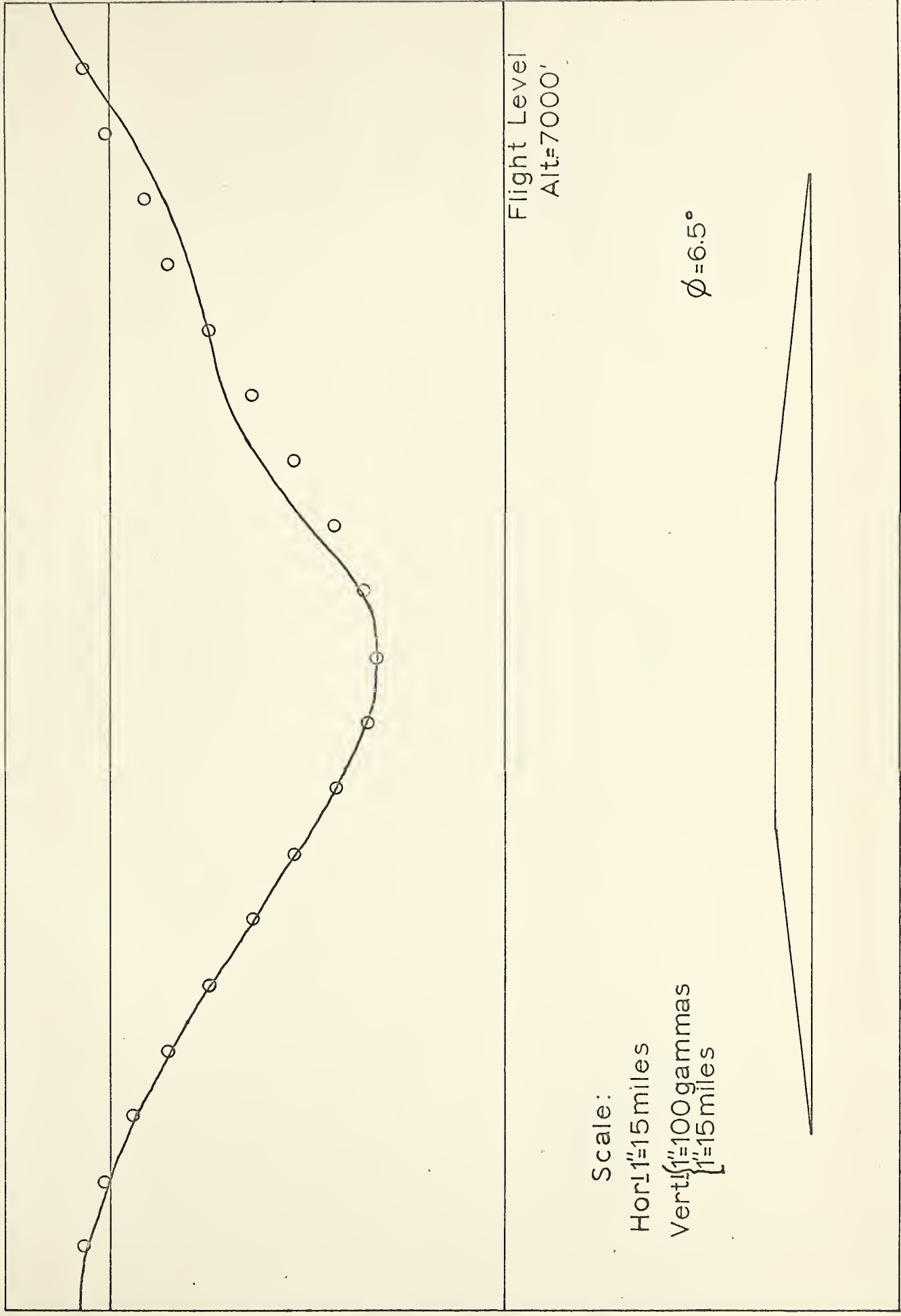
$\phi=16.5^\circ$

ANOMALY 'A' FOR FLIGHT 2

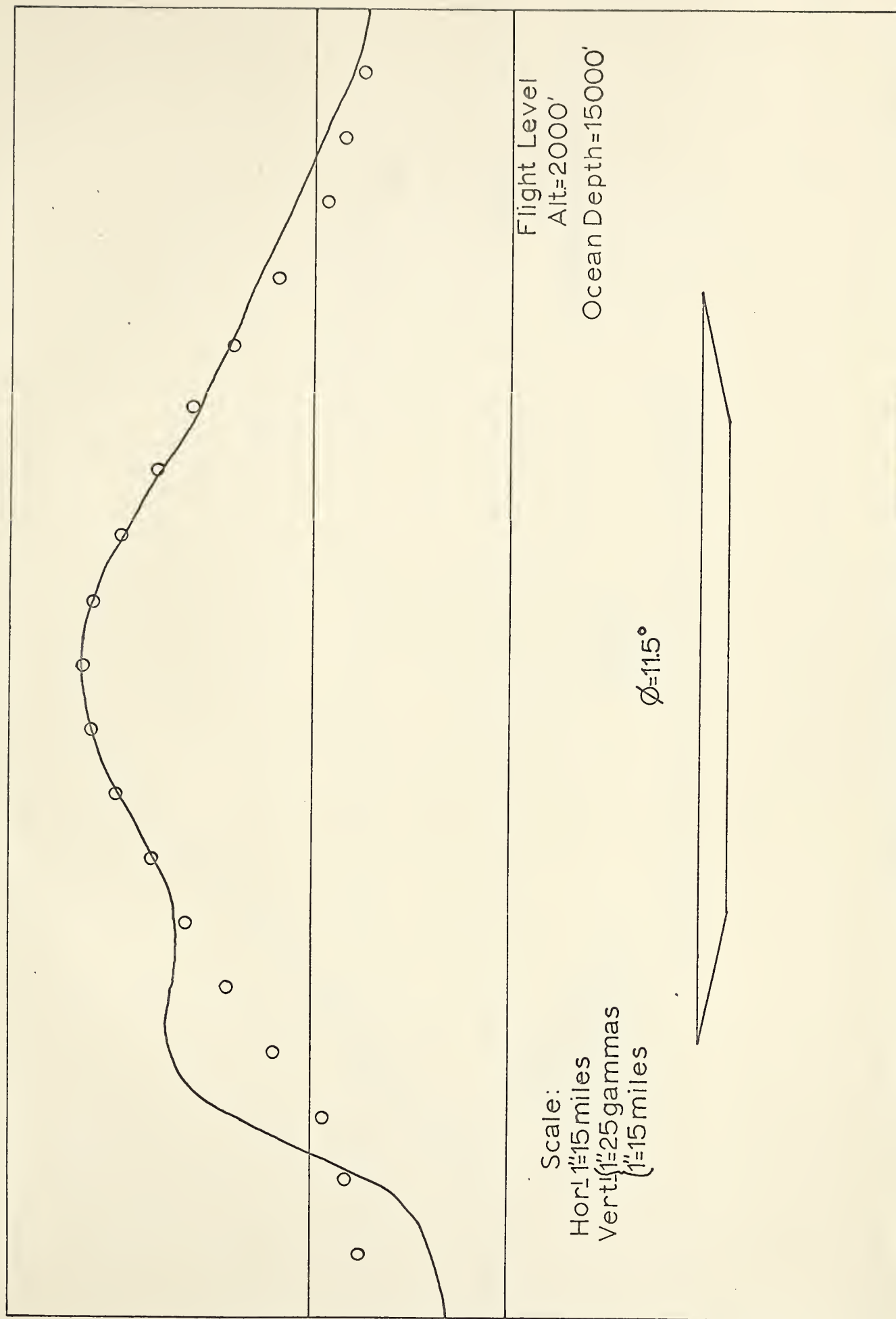
FIGURE 14



ANOMALY 'B' FOR FLIGHT 2

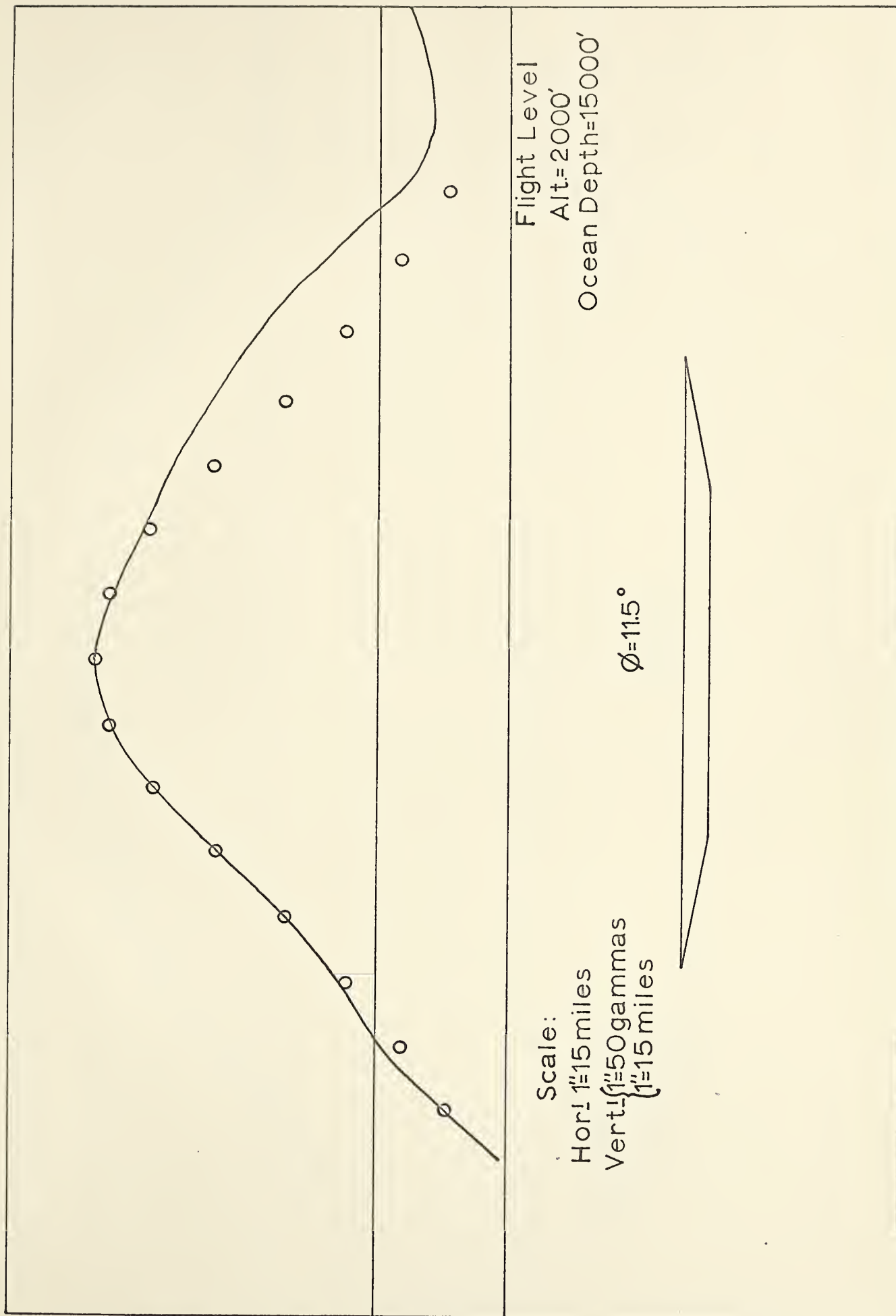


ANOMALY 'A' FOR FLIGHT 7



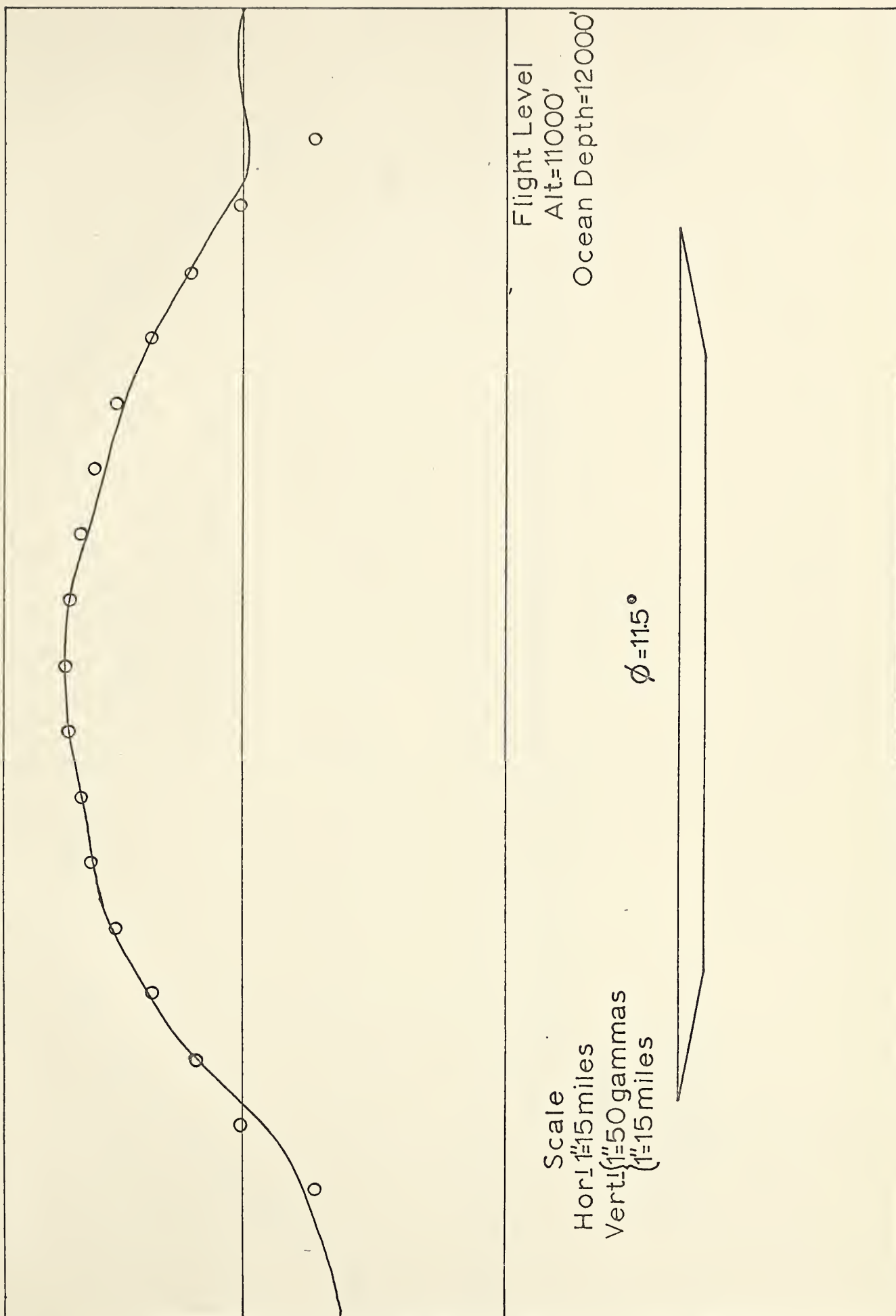
ANOMALY 'A' FOR FLIGHT 10

FIGURE 17



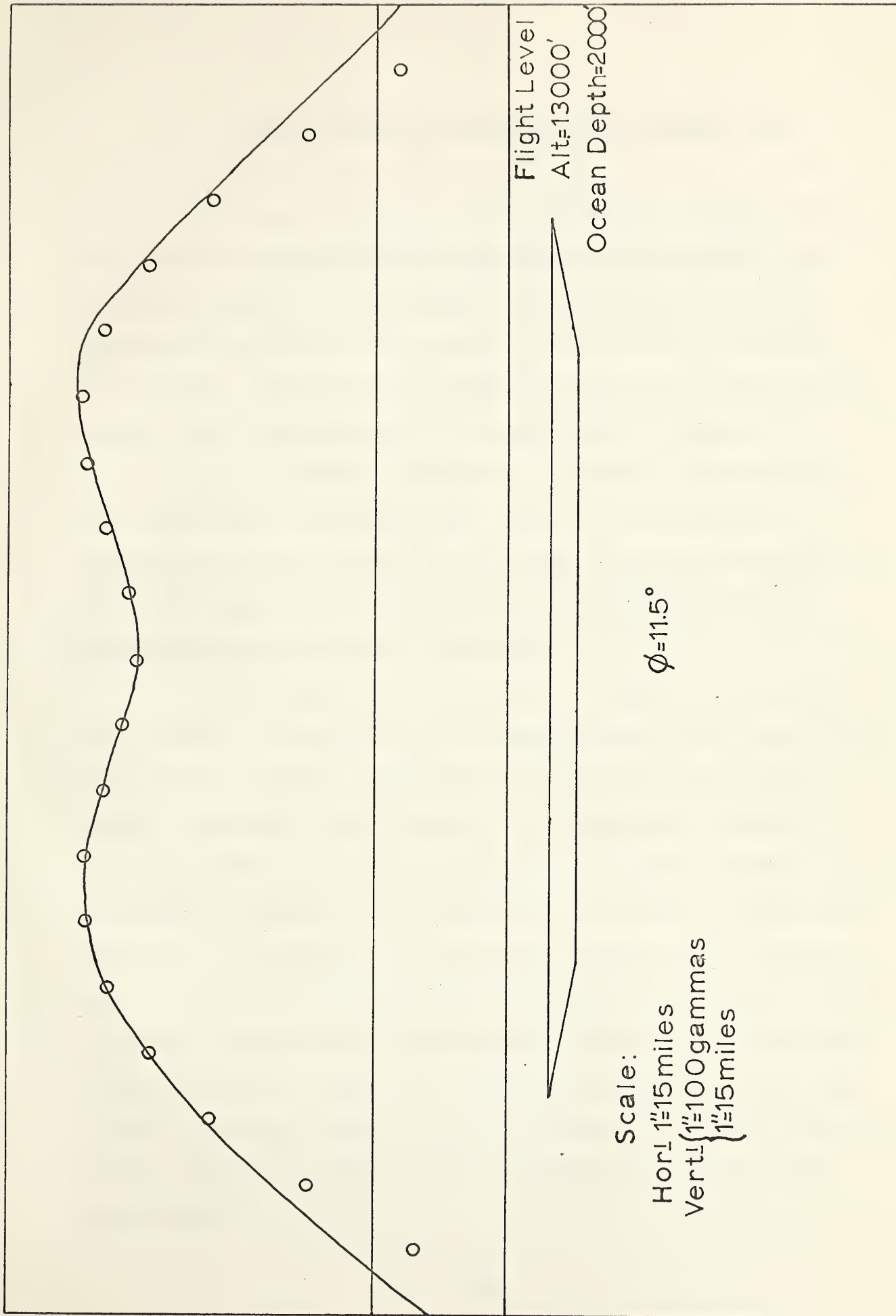
ANOMALY 'B' FOR FLIGHT 10

FIGURE 18



ANOMALY 'A' FOR FLIGHT 20

FIGURE 19



ANOMALY 'A' FOR FLIGHT 23

FIGURE 20

V. CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

In view of the fact that the amount of data available for the analyses and interpretations involved in this thesis was small, the conclusions made here must be termed suggestive as opposed to factual. In an effort to analyze the magnetic character of the earth's crust as a whole, one can see that a great amount of data would be required in order to form concrete conclusions. However, the results of this work agree favourably with those of the majority of research in this field and one can say that the interpretations set forth here are reasonably accurate and entirely feasible based on the data analyzed.

As pointed out in section 4.1 there is good agreement between the results of the power spectral analyses and those of the harmonic analyses of Alldredge and Van Voorhis (1961). In short, both indicate a non-magnetic character for the greater portion of the mantle. From the evidence of the weighted spectra for continental and oceanic regions one might say that spectral analysis might prove to be a useful tool in differentiating between continental and oceanic crusts in areas covered by ice, for example. However, the evidence is insufficient to indicate a definite difference in the amplitudes of peaks appearing on continental and oceanic spectra; further work along these lines may shed some light on this possibility.

In general, the Curie point depth estimates were deeper for the continental crust than those found by Vacquier and Affleck (11 - 15 miles, 1941). Moreover, the values found in the work of this thesis were in the range of 20 - 25 miles, which was the Curie point depth range predicted for magnetite by Birch (1955). Birch's estimates were based on calculations using temperature-depth curves for various models of the crust (see section 1.4). In order to check the hypothesis that the broad anomalies (periods > 200 miles) are due to anomalous features of the Curie point geotherm, one might investigate areas where anomalous heat flows have been recorded. In this way one could determine if the geotherm depth was greater in areas of lower heat flow and smaller for areas of high heat flow.

From an analysis of similar magnetic profiles Serson and Hannaford (1957) predicted susceptibilities $> 5 \times 10^{-3}$ c.g.s. for the oceanic crustal rocks and for the continental crustal rocks, $\geq 5 \times 10^{-2}$ c.g.s. The estimates made in section 4.2 agree with the above values for the oceanic crust but are smaller by a factor of 2 or 3 for the continental crust. The only depth, which might be interpreted as a Curie point geotherm depth, given by Serson and Hannaford, was that for the lower boundary of the layer of dipoles representing the oceanic magnetic crust. The depth given was ≥ 30 km (approximately 20 miles), which is in general agreement with the oceanic values of section 4.2. Along these channels, it might prove fruitful to carry out more accurate surveys

at altitudes of 8000 - 10,000 feet in order to provide more reliable estimates of the susceptibility contrasts at greater depths in the crust. In this way, one might obtain a better knowledge of the magnetic character of the rocks at these depths and hence a better understanding of the general nature of the rocks themselves.

In future, the following suggestions may prove helpful in planning surveys for long aeromagnetic profiles from which broad anomalies are to be extracted and interpreted:

- (i) maintain as high an accuracy as possible with respect to aircraft speed and navigation in order that anomalies can be accurately checked with surface and subsurface geology,
- (ii) survey those areas in which a good control for the dipole field trend can be obtained, if possible, to insure maximum accuracy for anomaly amplitudes,
- (iii) survey some of the chosen areas more than once to see if magnetic fluctuations affect the records drastically,
- (iv) fly global flights¹ (great circles), if possible, avoiding sharp turns, in order to investigate the magnetic properties of the core-mantle boundary.

¹Such flights are being made at present by project Magnet of the U. S. Hydrographic Office.

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APPENDICES

APPENDIX I

The Spectral Window

In order to recognize the need for the application of a spectral window in order to obtain a power spectrum for a set of discrete data, we shall first consider the continuous series rather than the discrete series.

Given a continuous record of finite length, it is clear that we cannot estimate the autocovariance function $C(\lambda)$ for arbitrarily long lags. Indeed, no estimate can be made for lags longer than the record. Thus, in place of

$$C(\lambda) = \lim_{X \rightarrow \infty} \frac{1}{X} \int_{-X/2}^{X/2} H(x) \cdot H(x+\lambda) dx$$

for all values of λ , we will have

$$\begin{aligned} c_{00}(\lambda) &= \frac{1}{X_n - |\lambda|} \int_{-(X_n - |\lambda|)/2}^{(X_n - |\lambda|)/2} H(x - \lambda/2) \cdot H(x + \lambda/2) dx \\ &= c_{00}(-\lambda) \end{aligned}$$

only for $|\lambda| \leq X_m < X_n$, where X_n is the length of the record, and X_m is the maximum lag to be used. $c_{00}(\lambda)$ will be called the "apparent autocovariance function", since its average value is $C(\lambda)$ for $|\lambda| \leq X_m$.

For the continuous series, the power spectral estimates are usually derived from a "modified apparent autocovariance function" by Fourier transformation.

While the modified apparent autocovariance functions, which are obtained by multiplying the apparent autocovariance function by suitable even functions of λ , are often far from being respectable estimates of the true autocovariance function, their transforms are very respectable estimates of "smoothed" values of the true spectral density.

Let $D_i(\lambda)$ be a prescribed even function of λ , subject to the restriction $D_i(0) = 1$, and $D_i(\lambda) = 0$ for $|\lambda| > X_m$, (where $i = 0, 1, 2, 3, 4$, depending upon the shape of $D_i(\lambda)$ for $|\lambda| < X_m$), and let the corresponding modified apparent autocovariance function be defined by

$$C_i(\lambda) = D_i(\lambda) \cdot C_{00}(\lambda)$$

$D_i(\lambda)$ may be regarded as a window of variable transmission which modifies the values of $C_{00}(\lambda)$ differently for different lags. It is therefore natural to call $D_i(\lambda)$ a "lag window."

It is clear that $C_i(\lambda) = 0$ for $|\lambda| > X_m$ although $C_{00}(\lambda)$ was not defined there. Because $C_i(\lambda)$ is defined for all values of λ , it has a perfectly definite Fourier transform $P_i(f)$, which should satisfy the relation,

$$P_1(f) = Q_1(f) * P_{00}(f)$$

where $Q_1(f)$ is the Fourier transform of $D_1(\lambda)$, the asterisk indicates convolution, and $P_{00}(f)$ is the Fourier transform of $C_{00}(\lambda)$. However, $P_{00}(f)$ is not determinate because $C_{00}(\lambda)$ is not specified for $|\lambda| > X_m$ (and its definition cannot be directly extended beyond $|\lambda| = X_n$). Nevertheless, since

$$\text{ave } [C_1(\lambda)] = D_1(\lambda) \cdot C(\lambda)$$

where $C(\lambda)$ is the true autocovariance function, it follows that

$$\text{ave } [P_1(f)] = Q_1(f) * P(f)$$

where $P(f)$ is the true power spectrum, that is, the Fourier transform of $C(\lambda)$. The average may be thought of as along space. This corresponds to replacing $H(x)$ by $H(x-\lambda)$, thus changing the stretch of $H(x)$ which is observed, and then averaging over λ . The corresponding relation,

$$\text{ave } [P_1(f_1)] = \int_{-\infty}^{\infty} Q_1(f_1-f) \cdot P(f) df$$

exhibits the average value of $P_1(f_1)$ as a smoothing (average-over-frequency) of the true power spectrum density $P(f)$ over frequencies "near" f_1 , with weights proportional to $Q_1(f_1-f)$. Then $P_1(f_1)$ may be said to be the collected impression of the true power spectrum $P(f)$ obtained through

a window of variable transmission $Q_1(f_1 - f)$. It is therefore natural to call $Q_1(f)$ the "spectral window" corresponding to the "lag window" $D_1(\lambda)$.

In the case of discrete series we again run into the problem of being unable to apply arbitrarily long lags. As a consequence we have to deal with finite Fourier series transformations rather than with infinite Fourier integral transformations, however the correspondence between multiplication and convolution persists.

The application of a spectral window in the discrete series case is slightly different. In the continuous case the window is introduced as a lag window before cosine transformation, in order to avoid transformation after convolution. In the discrete case, since convolution is not difficult, convolution after transformation is convenient, and the window is shaped after cosine transformation.

The reason for this change of procedure is that the $D_1(\lambda)$, for $i > 1$, are finite sums of cosines, so that their transforms are simply sums of spikes (Dirac delta-functions) at the appropriate spacing. Hence convolution becomes only smoothing with a set of weights whose values depend on the shape of the spectral window desired.

In choosing the proper lag window, one would like to concentrate the main lobe of $Q_1(f)$ near $f = 0$, keeping the side lobes as low as feasible. In order to concentrate

the main lobe one must make $D_1(\lambda)$, flat and rather blocky. In order to reduce the side lobes, however, one has to make $D_1(\lambda)$ smooth and gently changing. Since $D_1(\lambda)$ must vanish for $|\lambda| > X_m$ a compromise must be made. Such a compromise is described below.

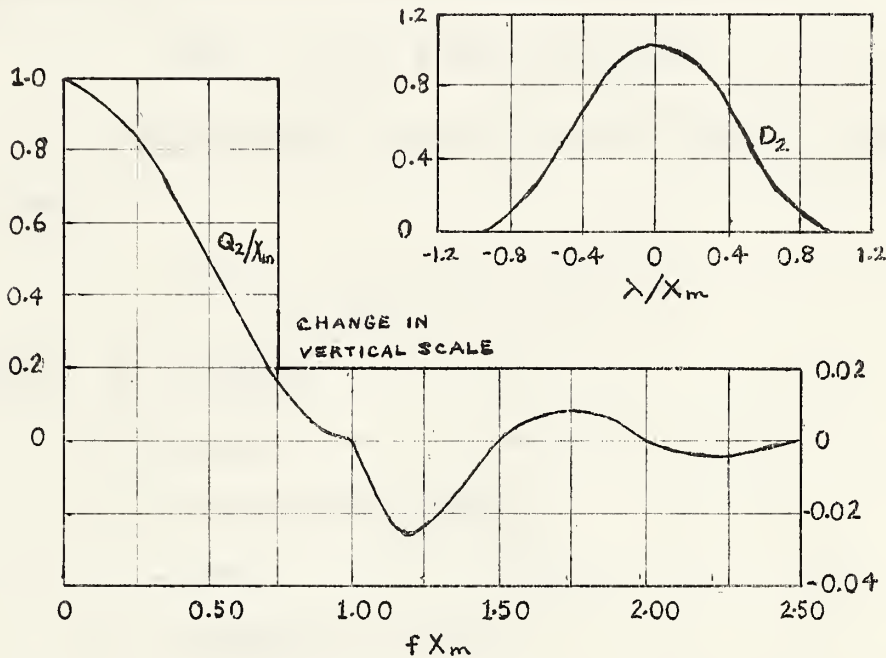


Figure 1

Figure 2

The lag window shown in figure 1 has the form

$$D_2(\lambda) = \frac{1}{2} \left(1 + \cos \frac{\pi \lambda}{X_m} \right) \quad |\lambda| < X_m$$

$$= 0 \quad |\lambda| > X_m$$

and the corresponding spectral window, shown in figure 2, is written

$$Q_2(f) = \frac{1}{2} Q_0(f) + \frac{1}{4} \left[Q_0\left(f + \frac{1}{2X_m}\right) + Q_0\left(f - \frac{1}{2X_m}\right) \right]$$

Hence the convolution with this particular spectral window means smoothing with weights 0.25, 0.5, 0.25. This form of smoothing is known as "hanning", after the Austrian meteorologist Julius von Hann.

APPENDIX II

Numerical Calculation of the Convolution Integral

The convolution of two functions is accomplished by means of the convolution integral given as,

$$Y(\delta) = \int_{-\infty}^{\infty} W(x) X(\delta-x) dx \quad \dots\dots\dots (1)$$

For discrete data the above integral degenerates to a sum,

$$Y(\delta) = \sum_{\sigma=1}^N W(\sigma) X(\delta-\sigma) \Delta\sigma \quad \dots\dots\dots (2)$$

where Y - output
 X - input
 W - weighting function
 δ - lag
 $\Delta\sigma$ - sampling interval

The equation (2) is easily programmed for the digital computer since it may be written as,

$$\begin{aligned} Y_1 &= W_1 X_1 \Delta\sigma \\ Y_2 &= (W_1 X_2 + W_2 X_1) \Delta\sigma \\ &\cdot \\ &\cdot \\ &\cdot \\ Y_N &= (W_1 X_N + W_2 X_{N-1} + \dots + W_{N-1} X_2 + W_N X_1) \Delta\sigma \end{aligned}$$

that is, the weighting moves from left to right on the profile, moving one sampling interval for each computation of one output point.

APPENDIX III

Calculation of Theoretical Anomalies Due to Warping of the
Curie Point Geotherm

These calculations were made assuming

- (i) a distribution of magnetic charge induced by the earth's field on the surface of a two-dimensional structure,
- (ii) polarization of the structure in the vertical direction only.

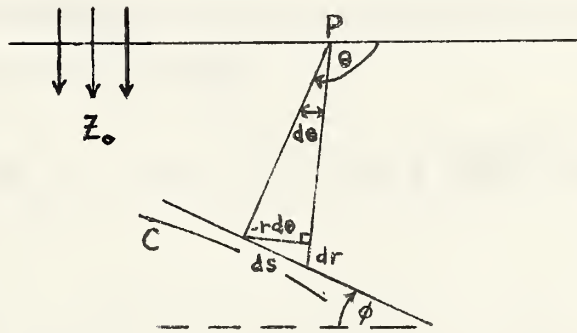


Figure 1

From figure 1, the potential at the point P, due to the line element of magnetic charge ds , is given by

$$2 k Z_0 \cos \phi \cdot \log_e r ds$$

where Z_0 is the vertical component of the earth's field, ϕ is the angle of inclination of the line element ds , and k is the magnetic susceptibility contrast for the structure.

The field at P due to the line element is

$$(2 k Z_0 \cos \phi \, ds)/r$$

and the vertical component of this field is

$$2 k Z_0 \cos \phi \frac{\sin \theta}{r} \, ds$$

where r and θ are the magnitude and direction of the radius vector from the point of observation, P, to the line element ds .

To obtain the expression for the field due to the entire line distribution with inclination ϕ we must integrate over the contour C, giving,

$$\Delta Z = 2 k \int_C (Z_0 \cos \phi \frac{\sin \theta}{r}) \, ds$$

From figure 1,

$$ds \sin \theta = -r d\theta \cos \phi + dr \sin \phi$$

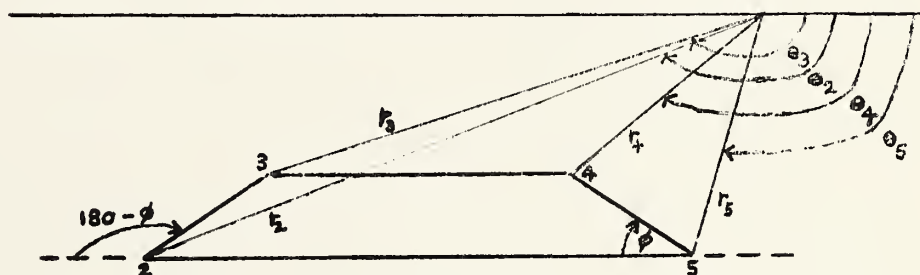
$$\therefore \Delta Z = 2 k \int_C Z_0 \cos \phi [\sin \phi (dr/r) - \cos \phi \, d\theta] \dots (1)$$

To obtain the field component for the entire structure the contour integral must be carried out over the entire cross section of the structure. If we represent the cross section by a sequence of line segments, one of which is shown in figure 1, (1) can be integrated over each segment, and the results can be summed over all segments to give

$$\Delta Z = 2 k \sum_i Z_0 \cos \phi_i [\cos \phi_i (\theta_i - \theta_{i+1}) + \sin \phi_i \log_e (\frac{r_{i+1}}{r_i})] \dots (2)$$

where θ_T = angle suspended by the top horizontal line segment
 θ_B = angle suspended by the bottom horizontal line segment.

Similarly, one may represent an upwarping of the geotherm by the following two-dimensional structure:



$$\theta_T = \theta_3 - \theta_4$$

$$\theta_B = \theta_2 - \theta_5$$

Upon applying the mathematical treatment used in the previous case, one obtains the following expression for the vertical component of the field due to the theoretical structure:

$$\Delta Z = 2 k Z_0 \left[\sin^2 \phi (\theta_T - \theta_B) + \sin \phi \cos \phi \log_e \frac{r_2 r_5}{r_3 r_4} \right]$$

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